

Non-Fermi liquid behavior in metallic quasicrystals with local magnetic moments

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30/Nov – 01/Dec 2015, Paris



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Jose Garcia (São Paulo)

Outline

- **Introduction**

- FL theory and NFL behavior
- Experiments in the $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal

- **Tiling models and quasicrystals**

- Geometrical and electronic properties

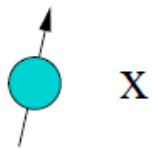
- **Kondo effect in metallic quasicrystals**

- Power-law distribution of Kondo temperatures
- NFL behavior

- **Conclusions**

Fermi-liquid (FL)

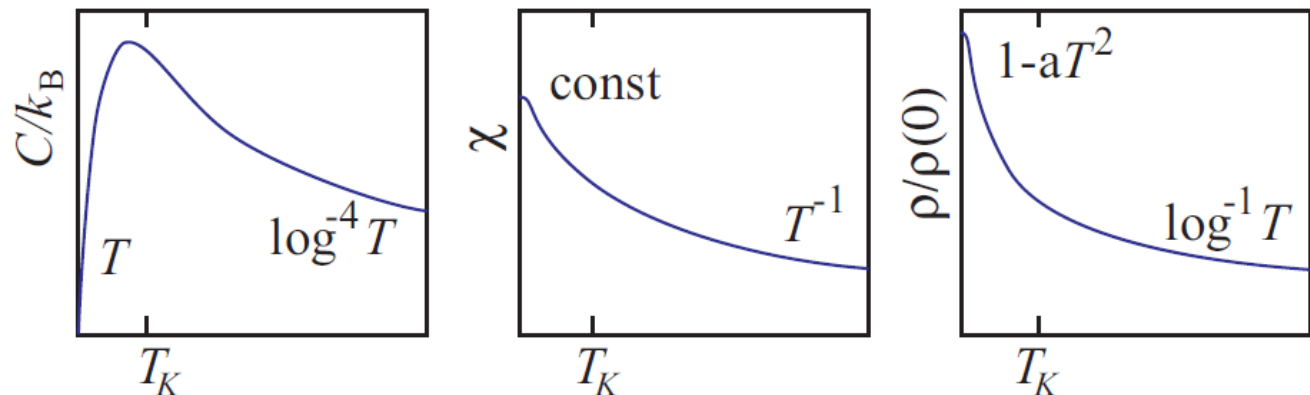
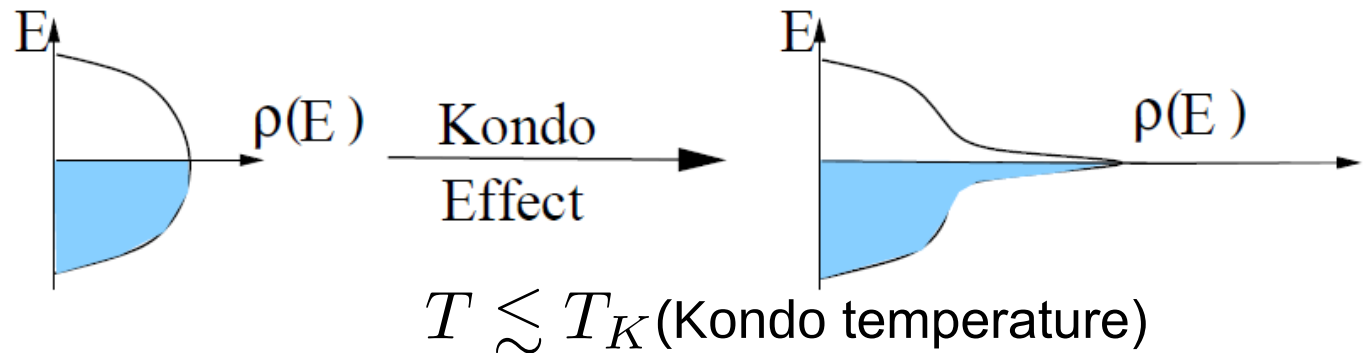
- Ground state of interacting fermions: akin to Fermi gas
- **Adiabaticity** + **Pauli principle**
- Metals at low-T, core of neutron stars, ^3He , Kondo problem...



Coleman, *Introduction to Many Body Physics*

Schofield, *Non-Fermi Liquids*

Nozières, J. Low Temp. Phys. **17**, 31 (1974)



Impurity contribution to C , χ , and ρ

Non-Fermi-liquid (NFL)

- FL known instabilities: Superconductivity, Magnetism, Band insulator, Mott insulator, Anderson insulator...
- Phase transitions or Metal-Insulator transitions: No adiabaticity
- Can the Fermi liquid fail within the **normal metallic state**?

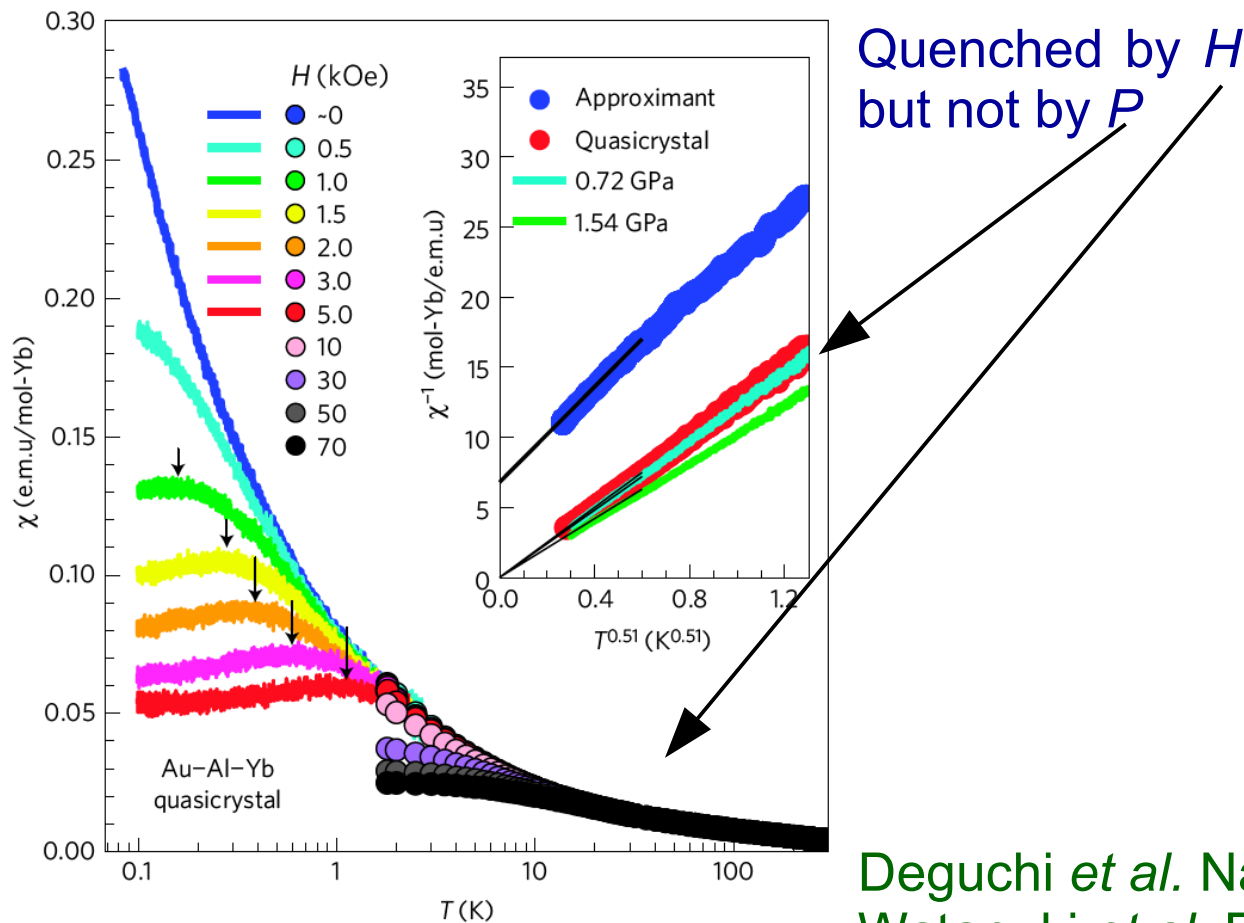
Yes! Known examples of *non-Fermi-liquid behavior*

- Metals in 1D: Luttinger liquid
- Two-channel Kondo models
- Metals close to a quantum critical point (QCP)
- Disordered Kondo models
- *Quasiperiodic Heavy Fermions*

NFL behavior in the $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal

- NFL without tuning any parameter!

$$\chi \propto T^{-0.51}$$



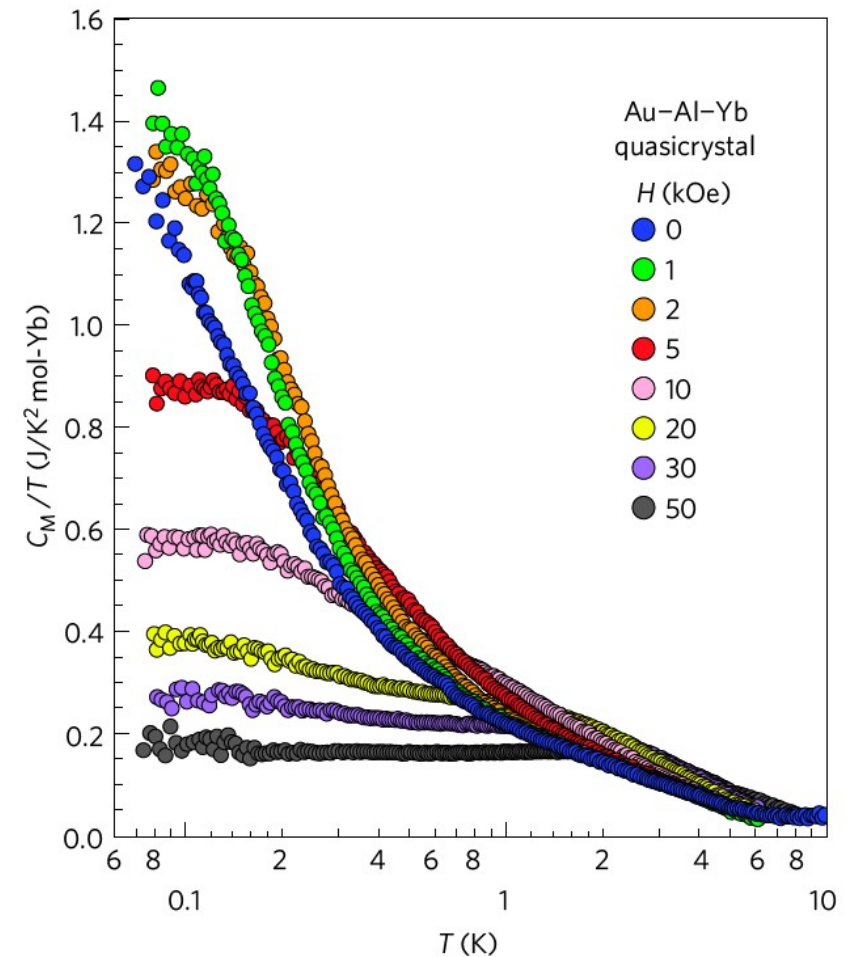
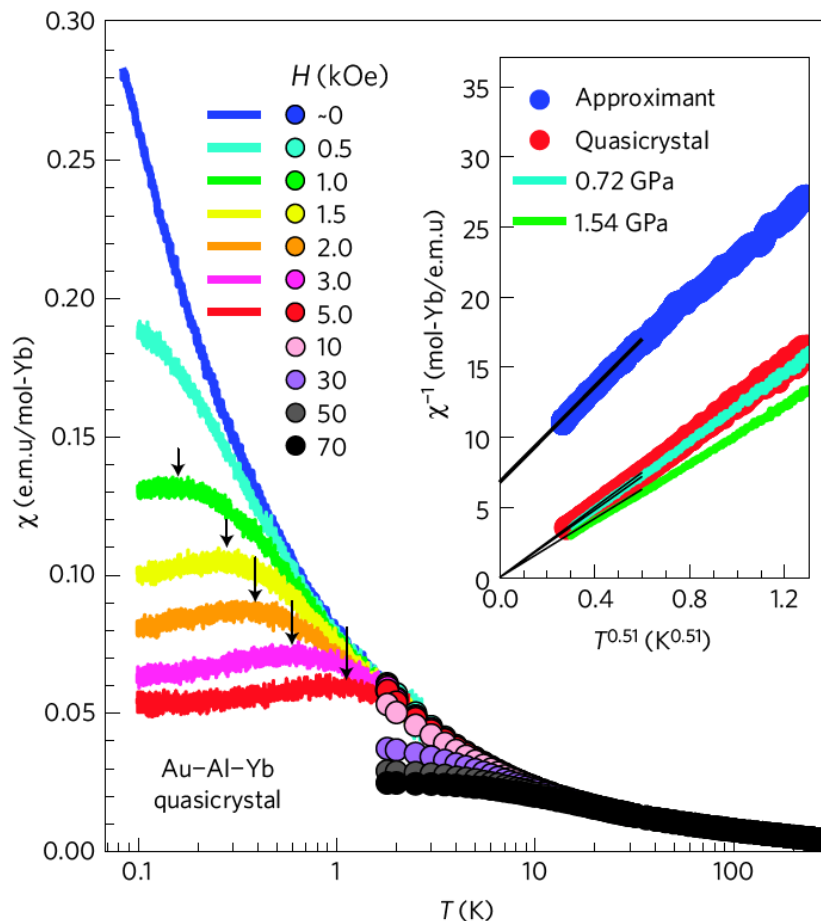
Deguchi *et al.* Nat. Mater. **11**, 1013 (2012)
Watanuki *et al.* PRB **86**, 094201 (2012)

NFL behavior in the $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal

- NFL without tuning any parameter!

$$C_M/T \propto -\log(T)$$

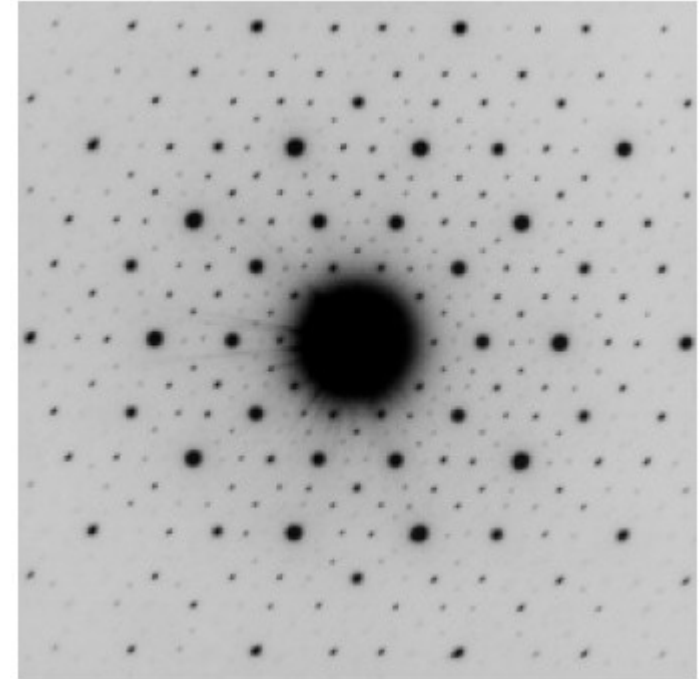
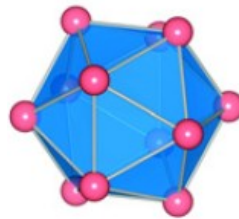
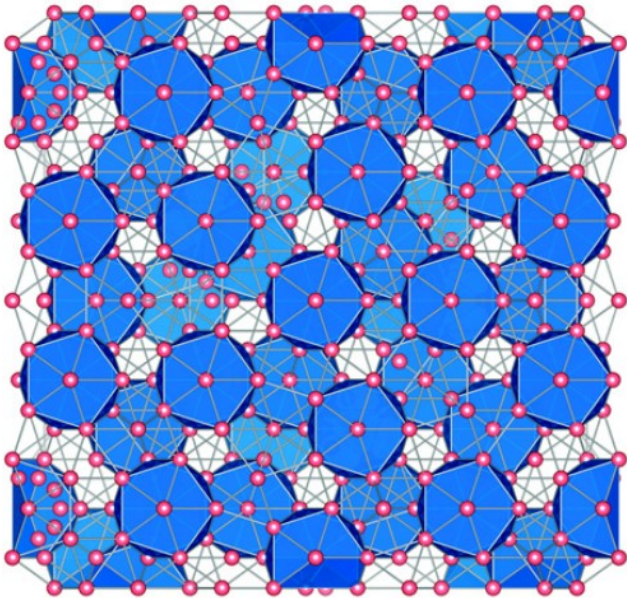
$$\chi \propto T^{-0.51}$$



Deguchi *et al.* Nat. Mater. **11**, 1013 (2012)
 Watanuki *et al.* PRB **86**, 094201 (2012)

$\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal

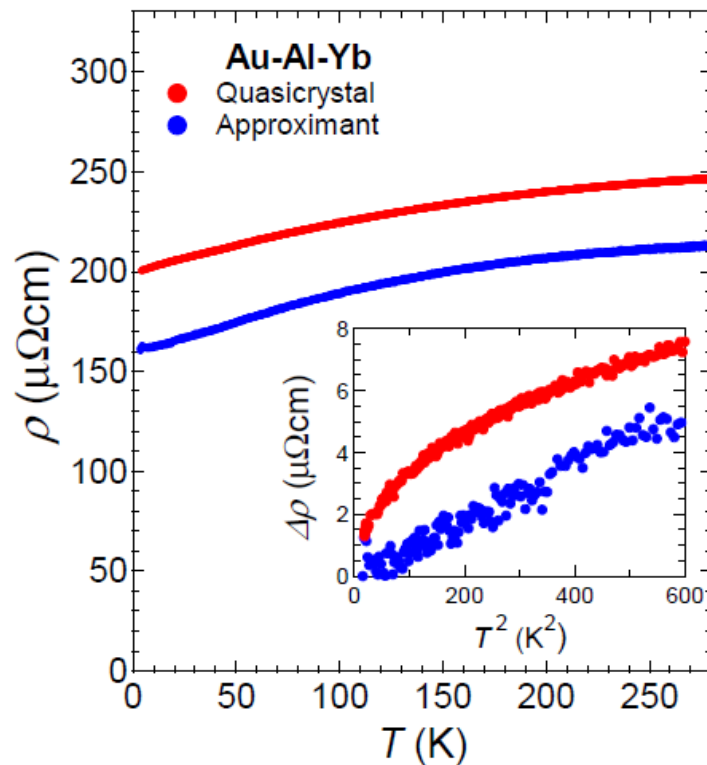
- $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ *quasicrystal*
- 10-fold symmetry diffraction pattern
 - 12 Yb icosahedron
- Icosahedron QC (Tsai-type)
 - Projected positions of Yb atoms



Ishimasa *et al.* Philos. Mag. 91, 4218 (2011)
Deguchi *et al.* Nat. Mater. **11**, 1013 (2012)
Watanuki *et al.* PRB **86**, 094201 (2012)

$\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal: Heavy Fermion

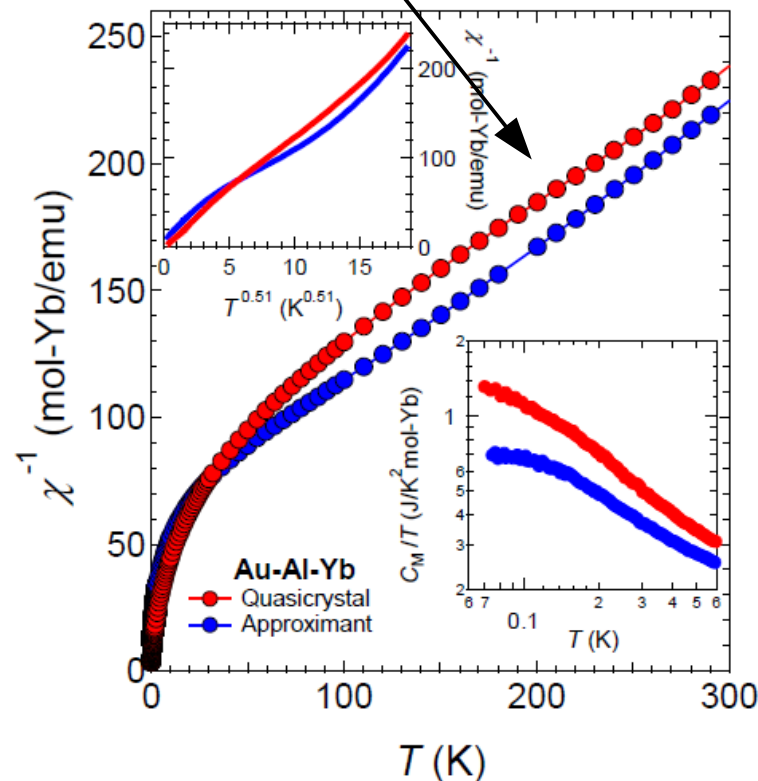
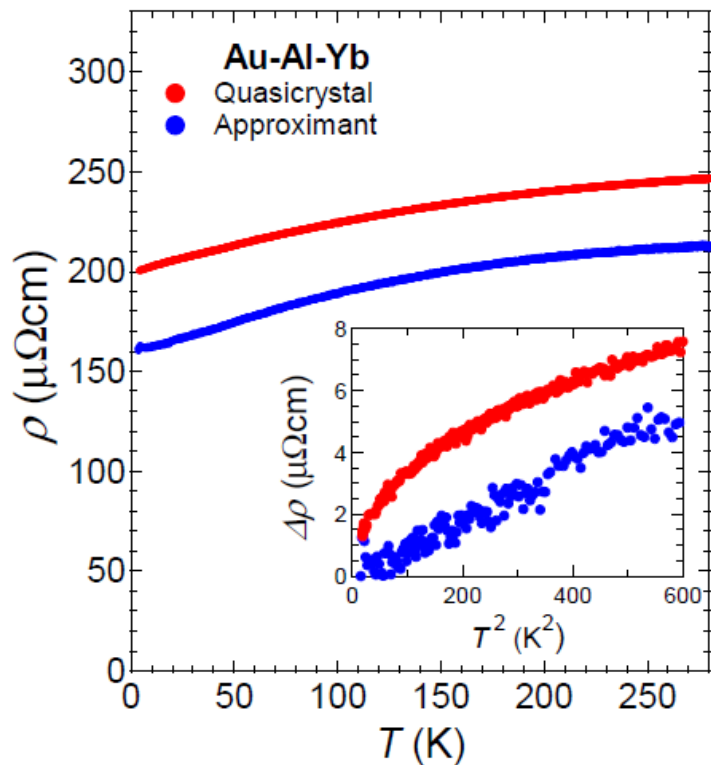
- *Metallic system* $\partial\rho(T)/\partial T > 0$



Deguchi *et al.*
Nat. Mater. **11**,
1013 (2012)

Au₅₁Al₃₄Yb₁₅ quasicrystal: Heavy Fermion

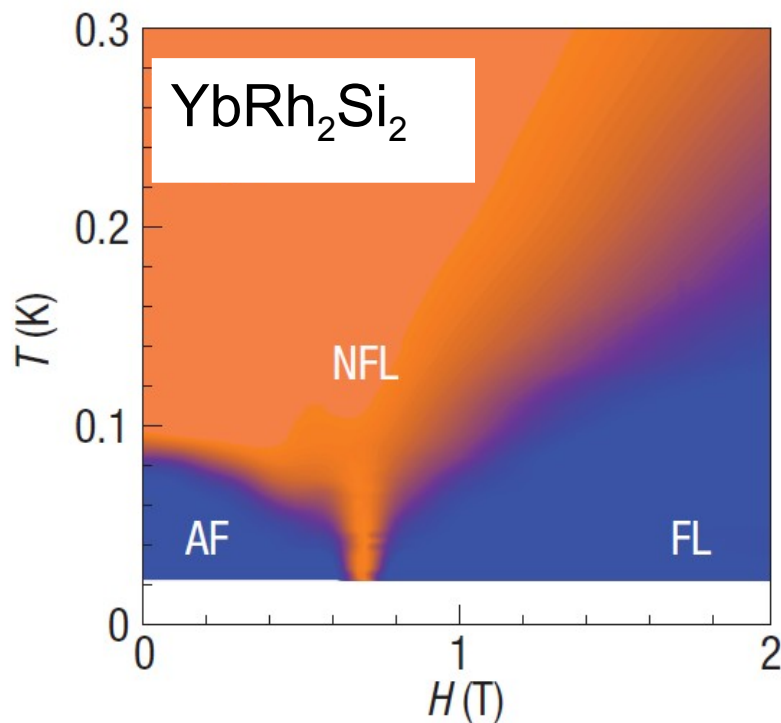
- **Metallic** system $\partial\rho(T)/\partial T > 0$
- **Local moments** at high-T: Curie law with $\mu = 3.92\mu_B$
- **Mixed-valence (2.61):** Yb²⁺ ($\mu = 0$) – Yb³⁺ ($\mu = 4.52\mu_B$)



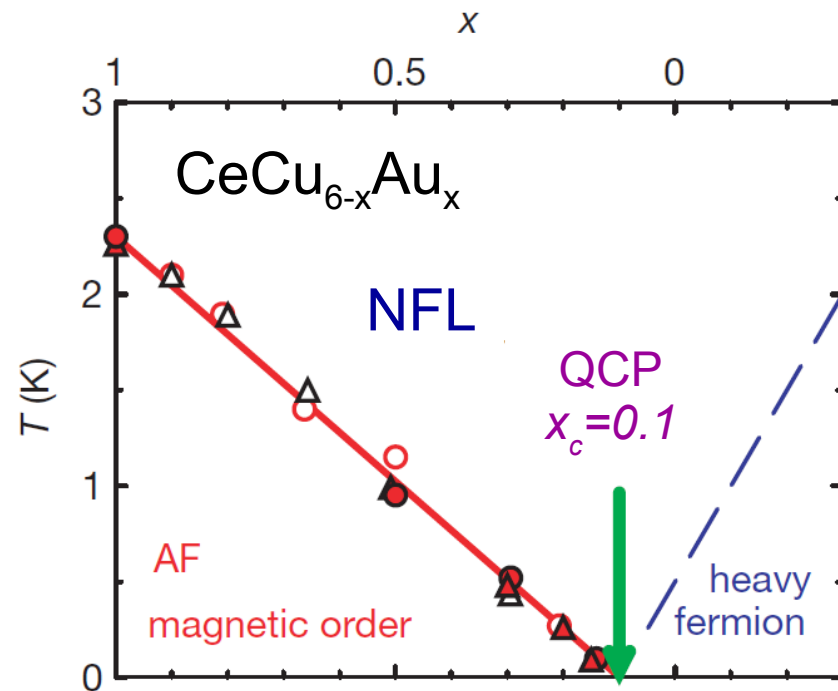
Deguchi *et al.*
Nat. Mater. **11**,
1013 (2012)

Quantum critical point scenario

- **NFL** behavior observed in several other **Heavy Fermions**
- Proximity to **Quantum Critical Point**
- **Tuning** of external parameter: field, doping, pressure, ...



Clusters *et al.*, Nature, **424**, 524 (2003)



Schröder *et al.*, Nature, **407**, 351 (2000)

Intrinsic NFL: $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal

- Conventional QCP approaches:

- Quantum valence criticality

Watanabe *et al.* J. Phys. Soc. Jpn. **82**, 083704 (2013)

- Fermion condensation quantum phase transition

Shaginyan *et al.* PRB **87**, 245122 (2013)

- *Parameter* driven QCP (pressure, doping, field): Fine-tuning

- Quasicrystalline environment of the light electrons considered only minimally

Our Goal: Study the Kondo problem in a quasicrystal

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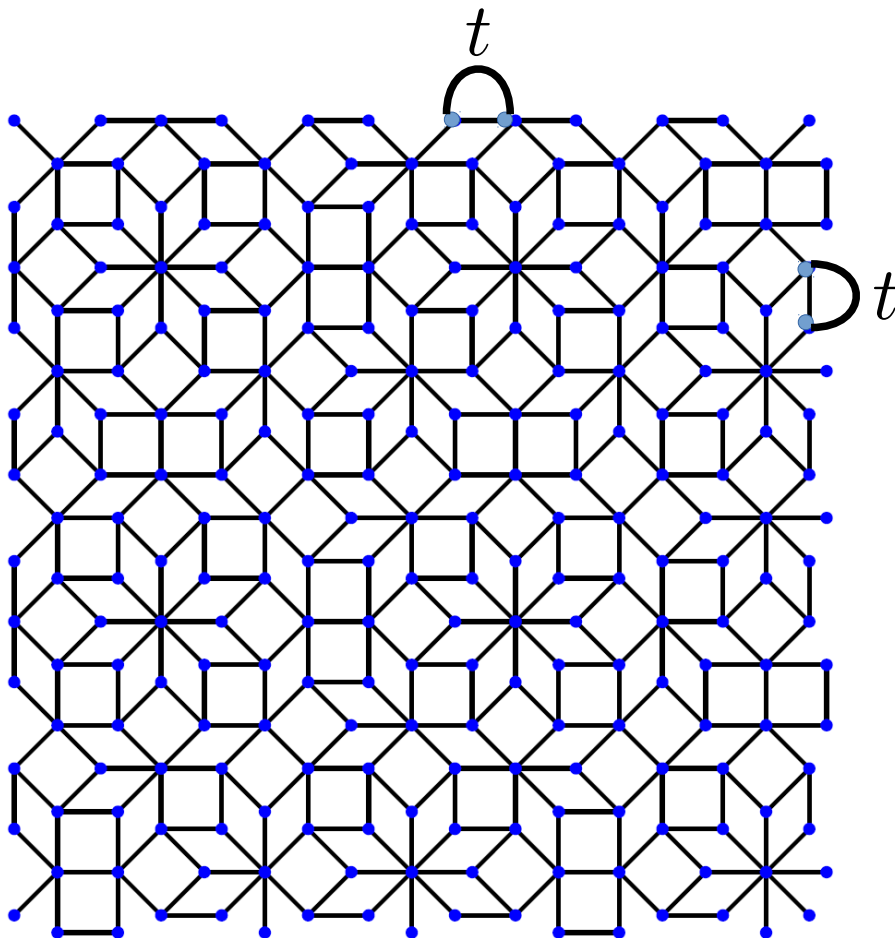
- Kondo effect in metallic quasicrystals

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- Conclusions

Octagonal tiling (Ammann-Beenker)

- Nearest neighbor tight-binding model. Non-interacting electrons hopping in a *quasiperiodic* potential



$$\mathcal{H}_c = -t \sum_{\langle ij \rangle, \sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right)$$

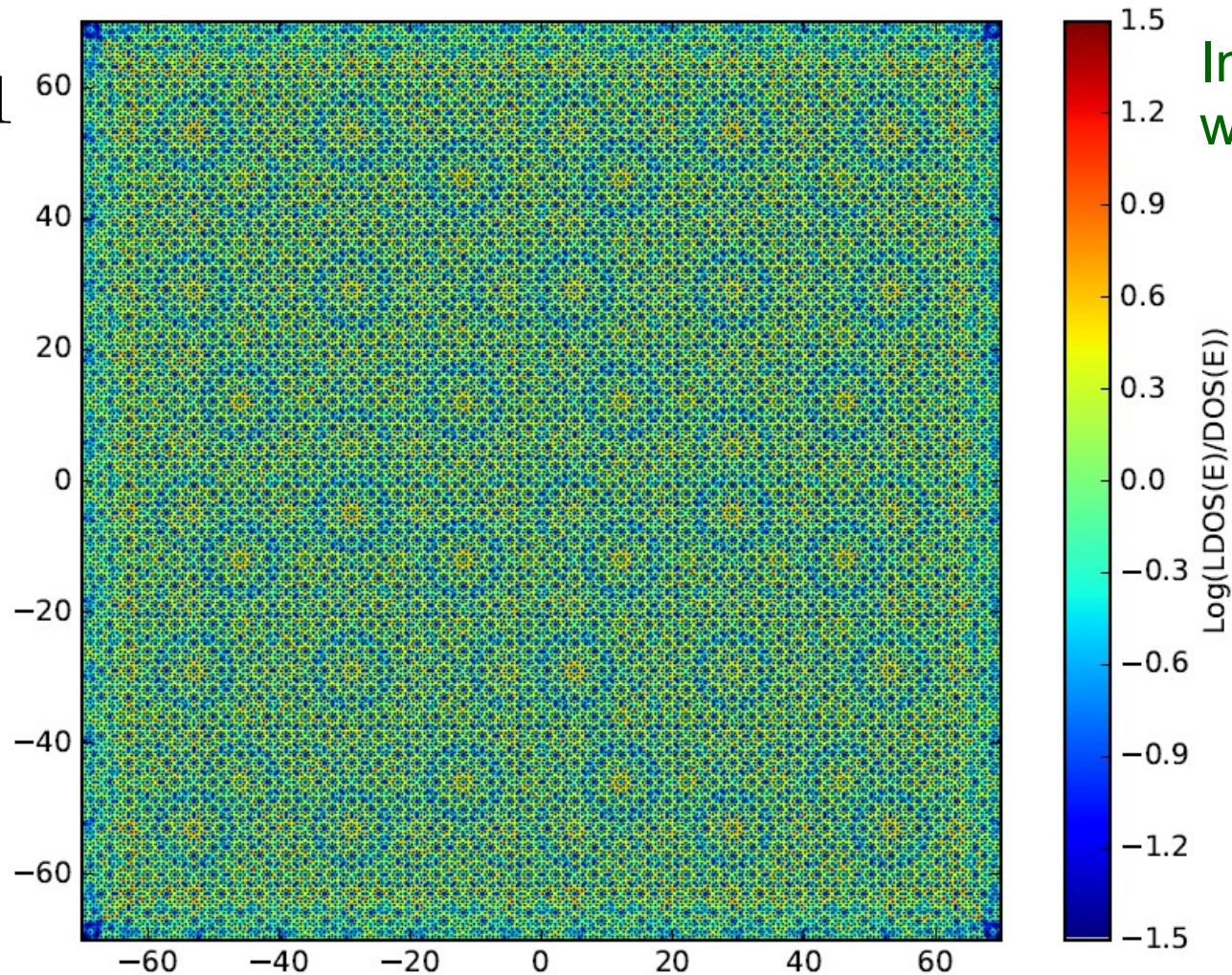
- Socolar, PRB **39**, 10519 (1989)
- Duneau, J. Phys. A **22**, 4549 (1989)
- Benza/Sire, PRB **44**, 10343 (1991)
- Grimm/Schreiber, in Quasicrystals—Structure and Physical Properties
- Jagannathan/Piéchon, Philos. Mag. **87**, 2389 (2007)
- ...

Octagonal tiling – Local density of states

- Unique for each different site.

$$N_a = 47321$$

$$\omega = -1.5t$$



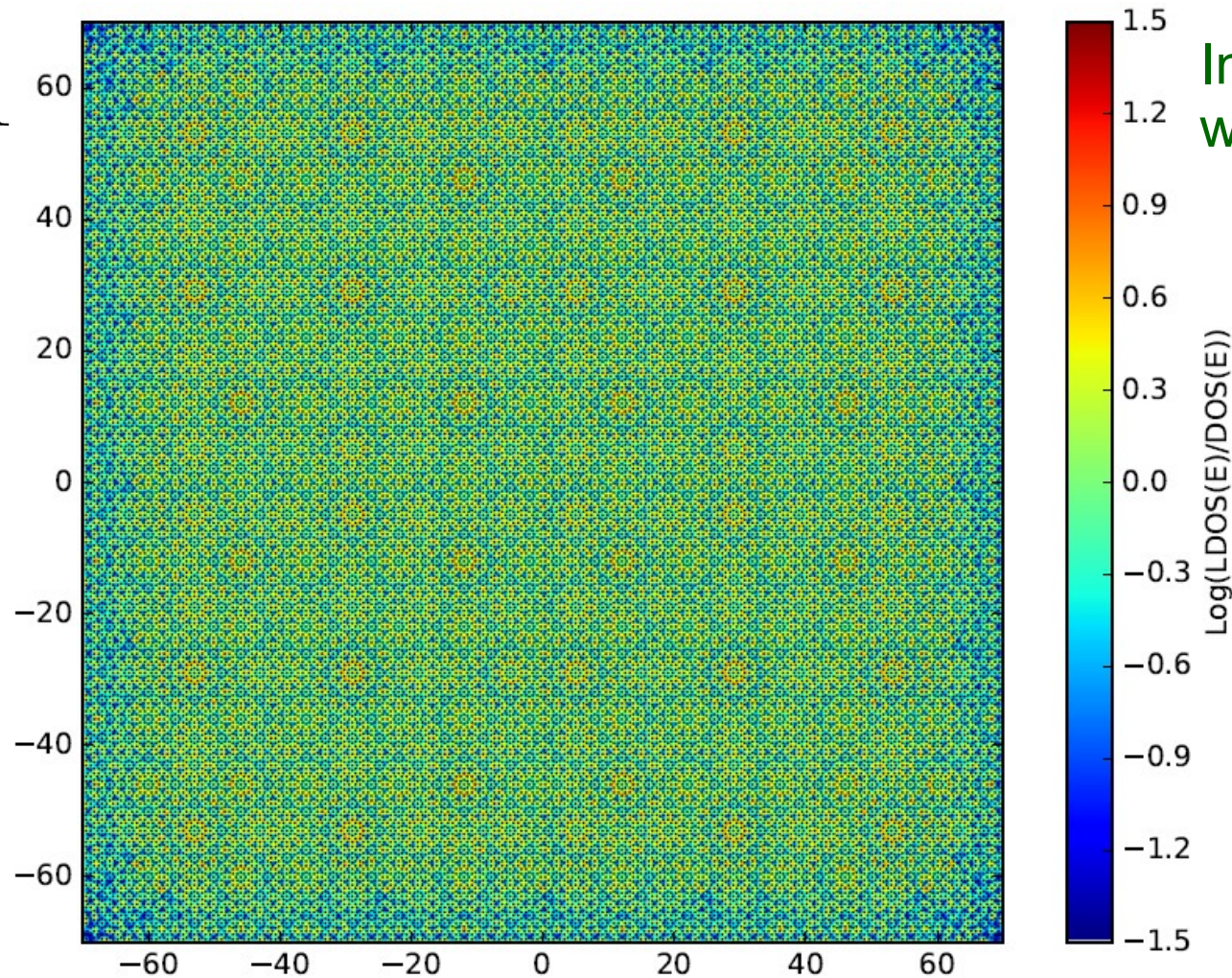
In collaboration
with Jose Garcia

Octagonal tiling – Local density of states

- Unique for each different site. Strong energy dependence

$$N_a = 47321$$

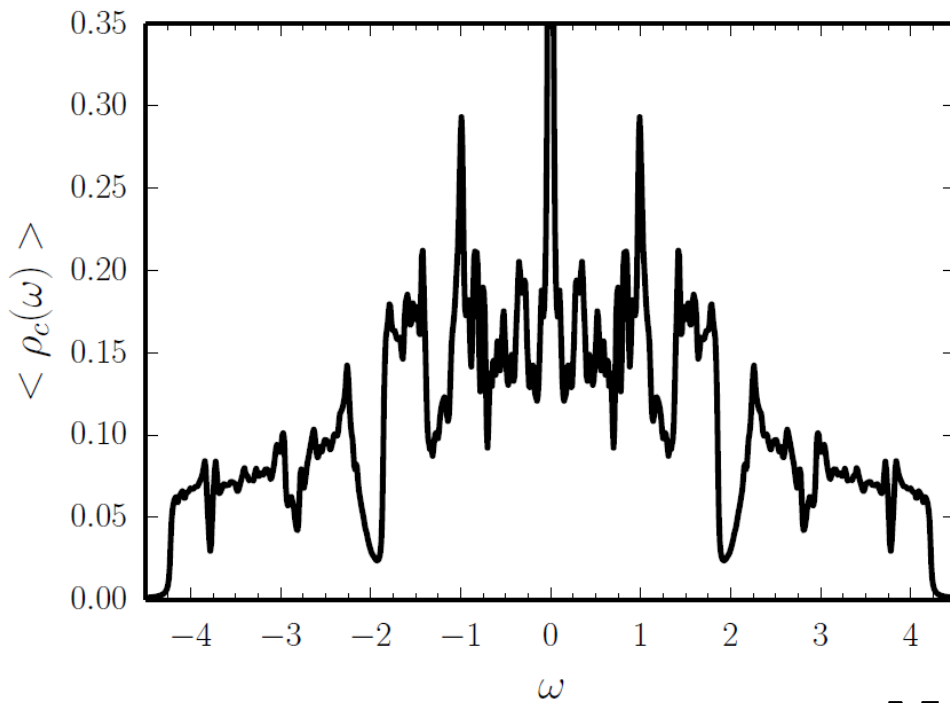
$$\omega = -2.2t$$



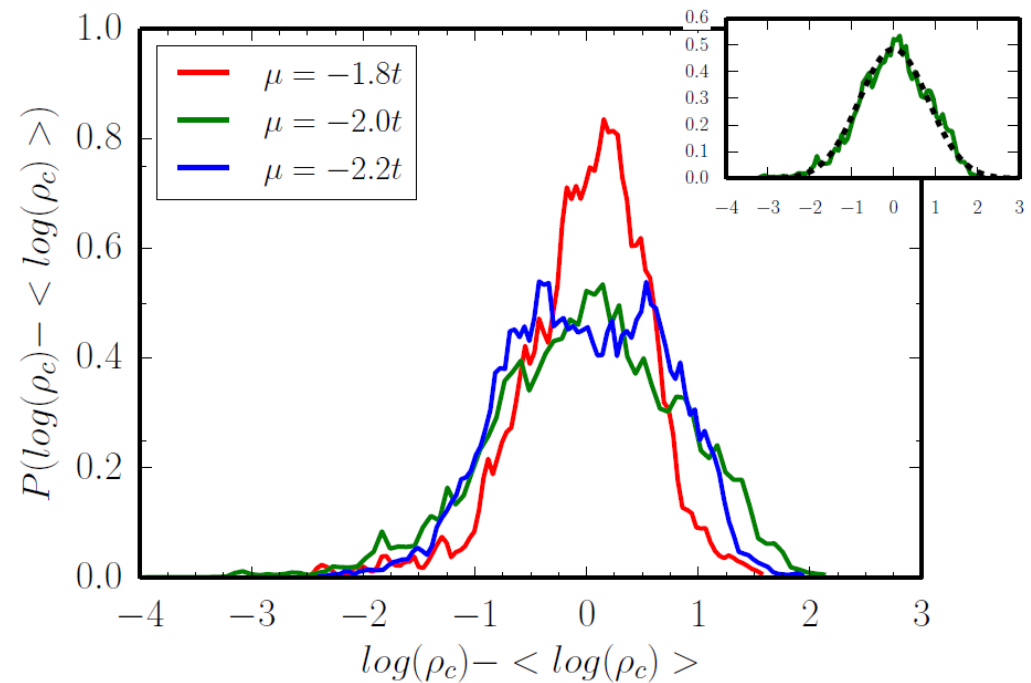
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Octagonal tiling – Density of states

- Averaged value (energy) and distribution (real space)
- Spiked in energy domain (averaged value over all sites)
- Approximately log-normal distribution in real space (fixed ω)



$$N_a = 8119$$



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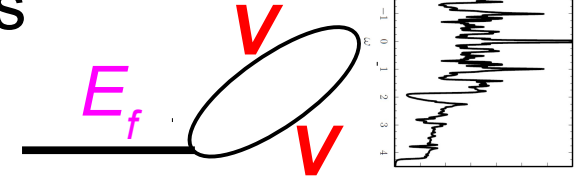
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Anderson impurity model (AIM)

- Localized f -orbital hybridizes with c -electrons
- $U \rightarrow \infty$ AIM: Mixed-valence ($n_f \leq 1$)
- Slave-boson MF approach



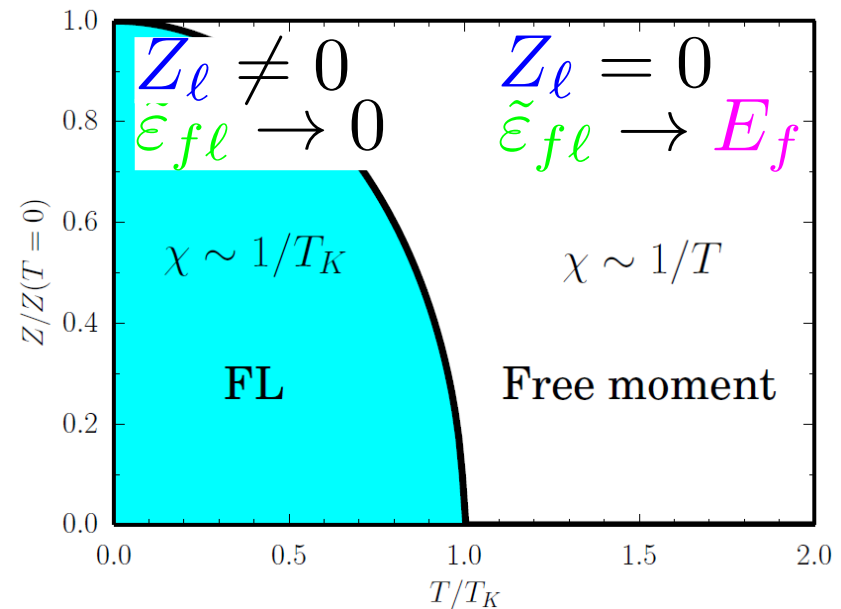
$$\mathcal{H} = \mathcal{H}_c + \tilde{\epsilon}_{f\ell} \sum_{\sigma} n_{f\sigma} + V \sqrt{Z_{\ell}} \sum_{\sigma} \left(f_{\ell\sigma}^{\dagger} c_{\ell\sigma} + c_{\ell\sigma}^{\dagger} f_{\ell\sigma} \right) + (\tilde{\epsilon}_{f\ell} - E_f) (Z_{\ell} - 1)$$

- Low-energy description

$$\tilde{V} = V \sqrt{Z_{\ell}}$$

$$T_K^{\ell} = D \exp \left[-1 / J \rho_{\ell}^c(0) \right]$$

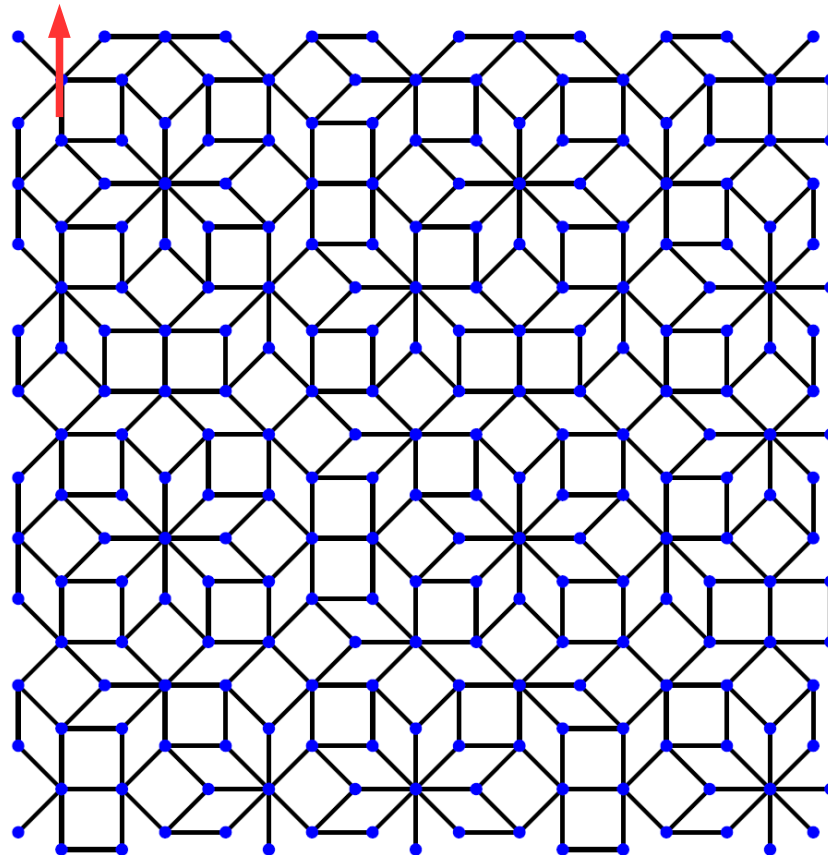
$$J = 2V^2 / |E_f|$$



Read/Newns, J. Phys. C **16**, L1055 (1983)
 Coleman, PRB **29**, 3035 (1984)

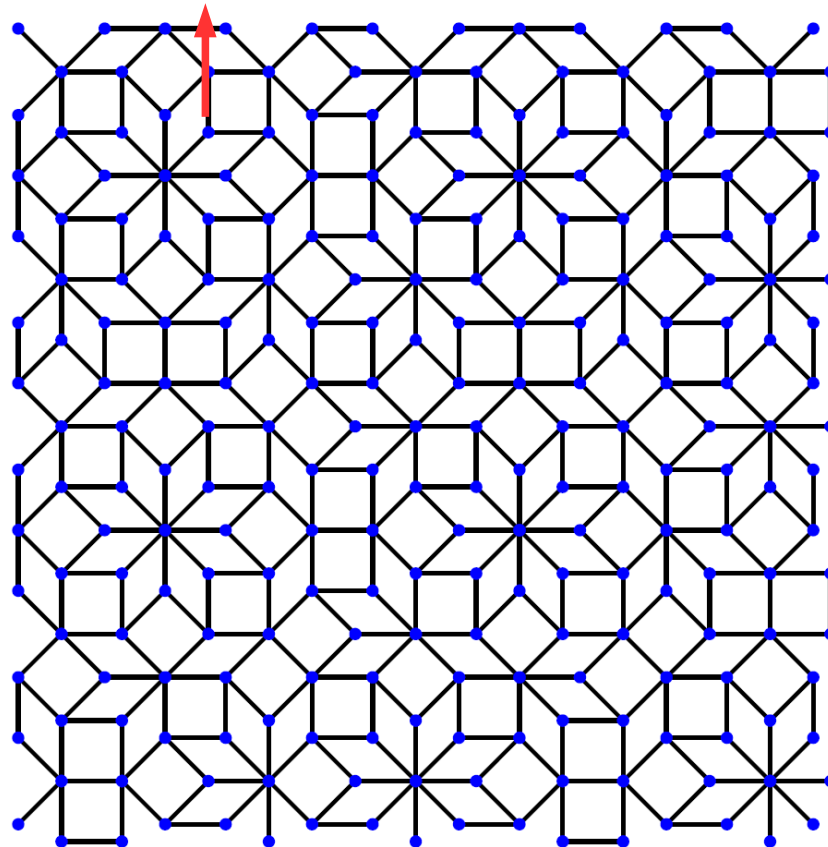
Kondo impurities in a quasicrystal

- Quasicrystal \rightarrow Different environments: $\Delta_{f\ell}(\omega) = V^2 G_{\ell\ell}^c(\omega)$
- **One Kondo impurity at each site.** N_a values of T_K : $P(T_K)$



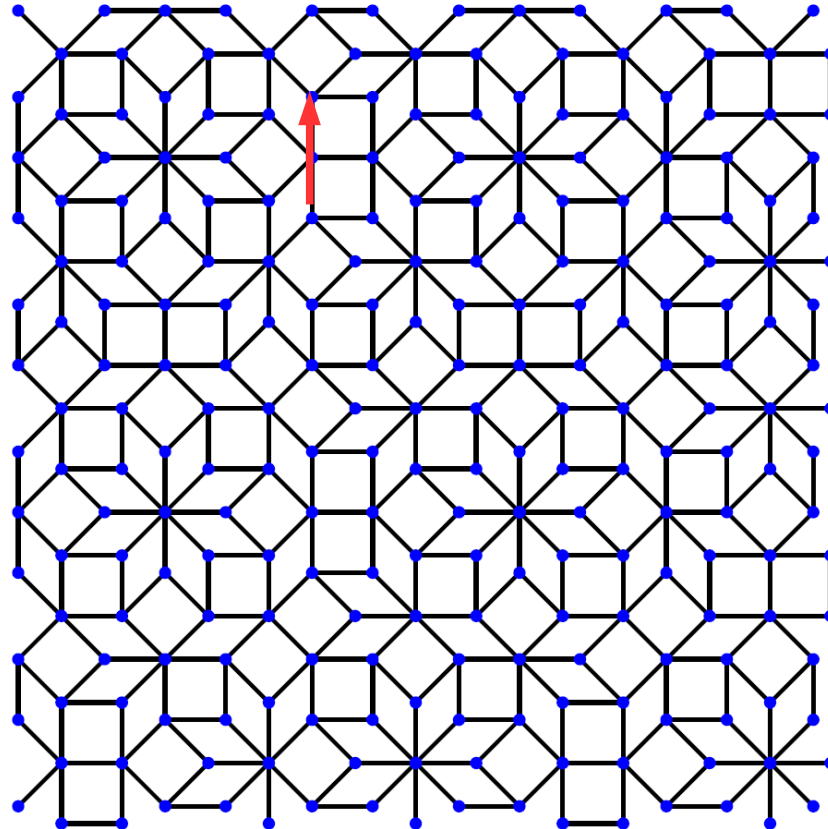
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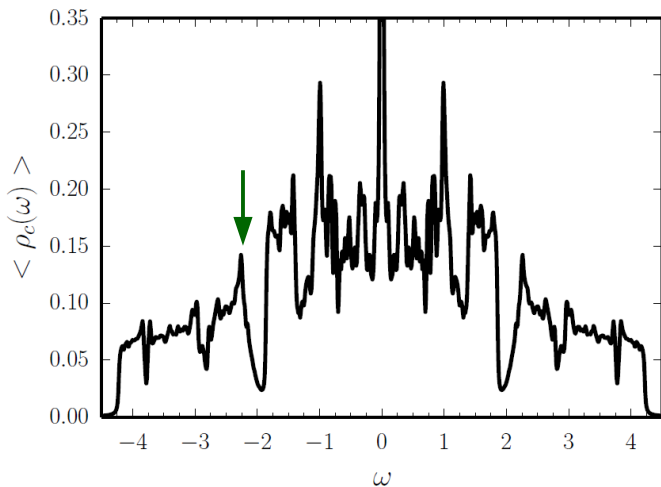
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Power-law distribution of Kondo temperatures

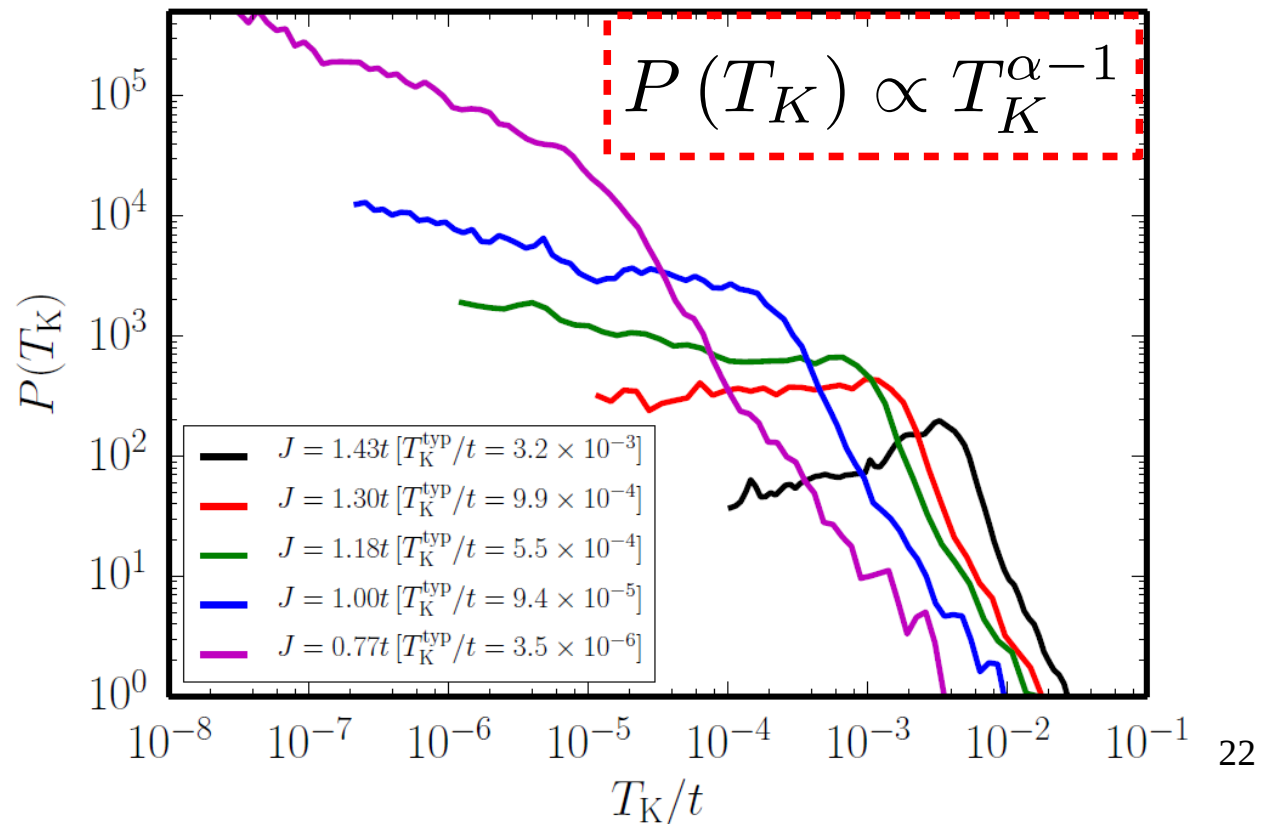
- Quasicrystal \rightarrow Different environments: $\Delta_{f\ell}(\omega) = V^2 G_{\ell\ell}^c(\omega)$
- **One Kondo impurity at each site**. N_a values of T_K : $P(T_K)$
- Remarkably, we get a **power-law distribution at low- T_K**



$$\mu = -2.2t$$

$$N_a = 1393$$

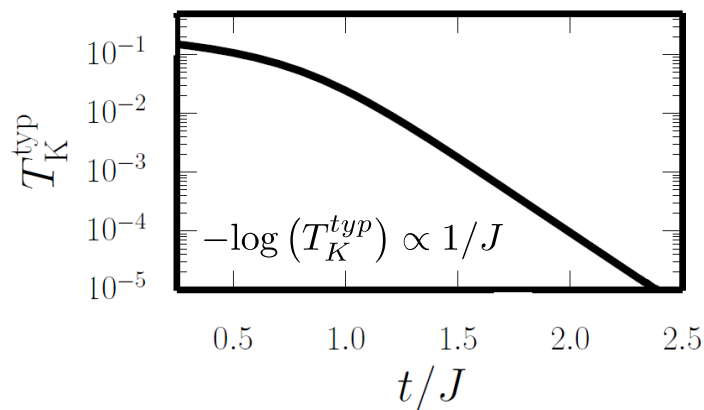
$$J = 2V^2 / |E_f|$$



Power-law distribution of Kondo temperatures

- Quasicrystal \rightarrow Different environments: $\Delta_{f\ell}(\omega) = V^2 G_{\ell\ell}^c(\omega)$
- **One Kondo impurity at each site**. N_a values of T_K : $P(T_K)$
- Remarkably, we get a **power-law distribution** below T_K^{typ}

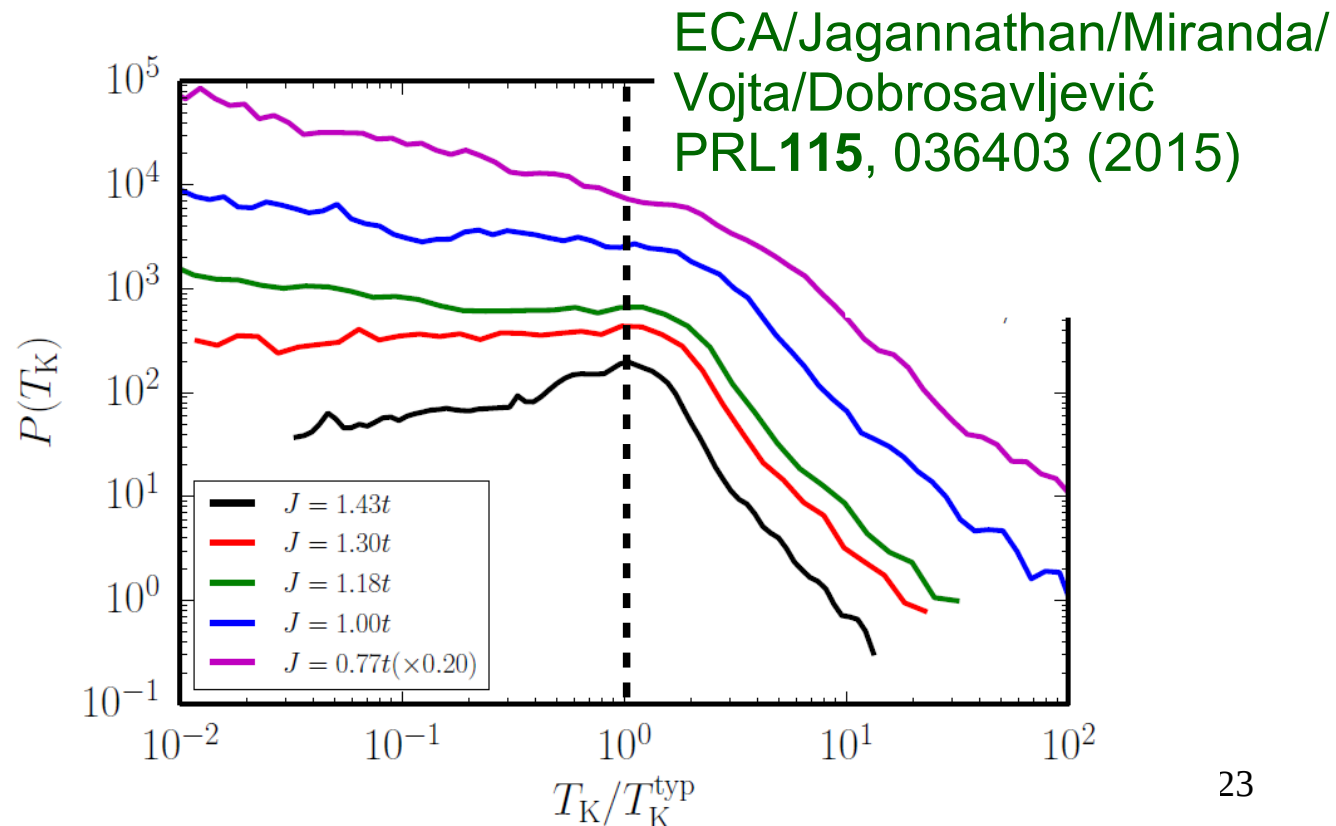
$$T_K^{typ} \equiv \exp[\langle \ln(T_K) \rangle]$$



$$\mu = -2.2t$$

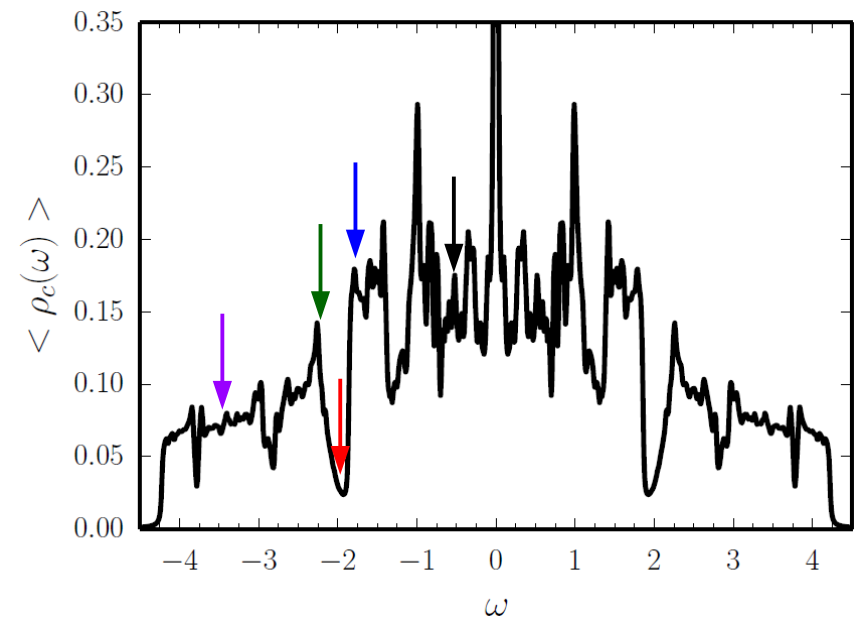
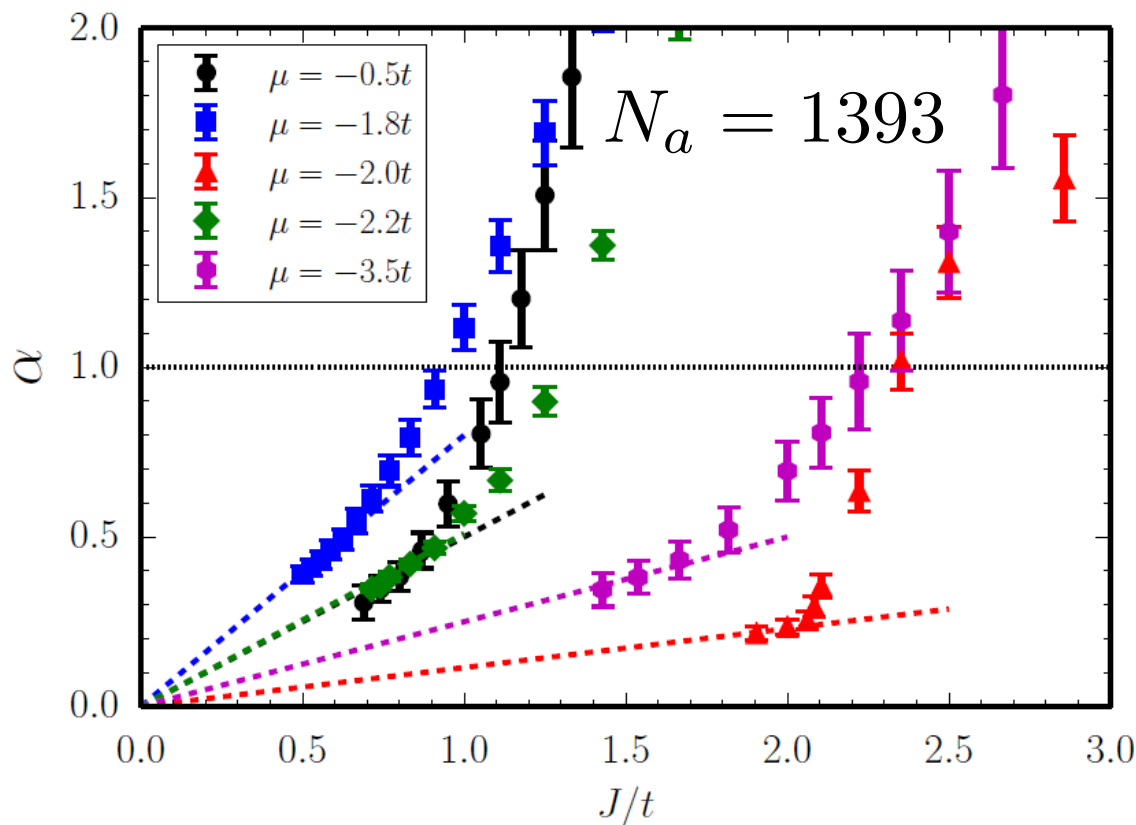
$$N_a = 1393$$

$$J = 2V^2 / |E_f|$$



Non-universal power-law exponent α

- $P(T_K) \propto T_K^{\alpha-1}$ for all μ , with $\alpha \propto J \langle \rho_c(0) \rangle$ as $J \rightarrow 0$
- As the DOS, α has huge energy dependence
- **NFL liquid behavior for $\alpha < 1$** . Unquenched spins: $T_K \rightarrow 0$



ECA/Jagannathan/Miranda/
Vojta/Dobrosavljević
PRL **115**, 036403 (2015)

Free spins at low-T: route to NFL

- Number of free local moments at a given T

$$n_{free}(T) = \int_0^T P(T_K) dT_K \sim \int_0^T T_K^{\alpha-1} dT_K \sim T^\alpha$$

Spins with $T_K < T$: essentially free

- Susceptibility

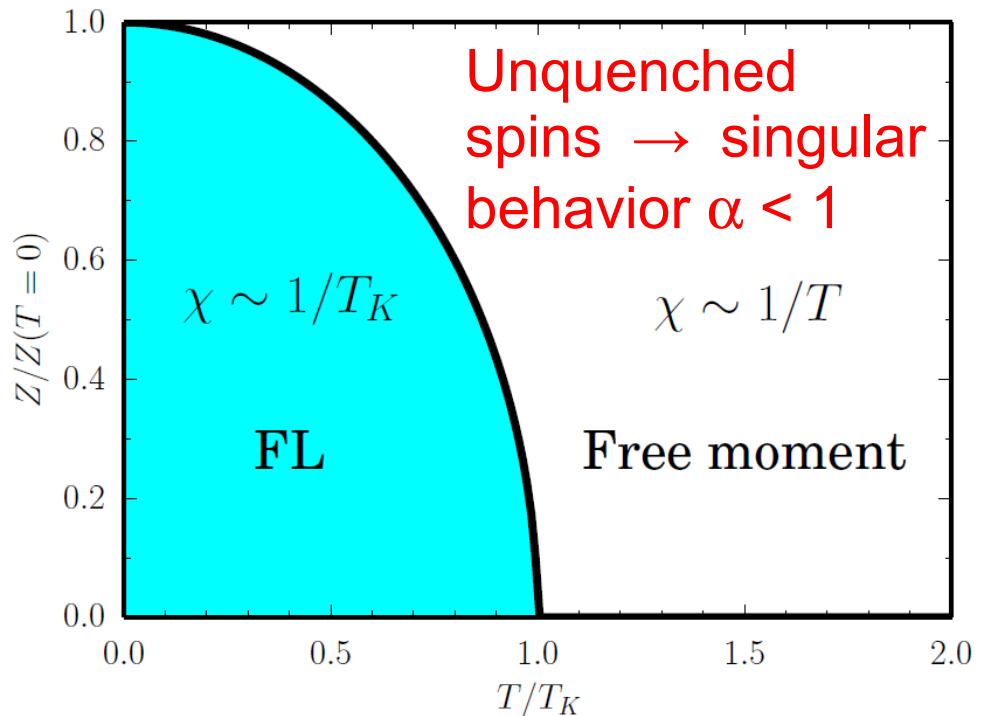
$$\chi(T) \sim n_{free}(T) / T \sim T^{\alpha-1}$$

- Entropy

$$S(T) \sim n_{free}(T) \ln 2 \sim T^\alpha$$

- Sommerfeld coefficient

$$\gamma(T) = \frac{C(T)}{T} = \frac{\partial S}{\partial T} \sim T^{\alpha-1}$$



NFL behavior – Free spins at $T \rightarrow 0$

- **Single energy scale:** $\begin{cases} T \lesssim T_K, & \text{Fermi-liquid} \\ T \gtrsim T_K, & \text{Free spin} \end{cases} \chi(T, T_K) = \frac{1}{T_K} f\left(\frac{T}{T_K}\right)$

- Averaged value of the single-impurity susceptibility

$$\langle \chi(T) \rangle = \int dT_K P(T_K) \chi(T, T_K) = \chi_r + \underbrace{\int_0^\Lambda dT_K T_K^{\alpha-1} \frac{1}{T_K} f\left(\frac{T}{T_K}\right)}_{\propto T^{\alpha-1}}$$

Regular + Singular

- Miranda/Dobrosavljević/Kotliar, J. Phys. Cond. Mat **8**, 9871 (1996)
- Rappoport/Boechat/Saguia/Continentino, EPL **61**, 831 (2003)
- Cornaglia/Grempel/Balseiro, PRL **96**, 117209 (2006)
- Kettemann/Mucciolo/Varga, PRL **103**, 126401 (2009)
- Miranda/Dias da Silva/Lewenkopf, PRB **90**, 201101 (2014)
- ...

$$\chi(T) \sim N(T)/T \propto T^{\alpha-1}$$

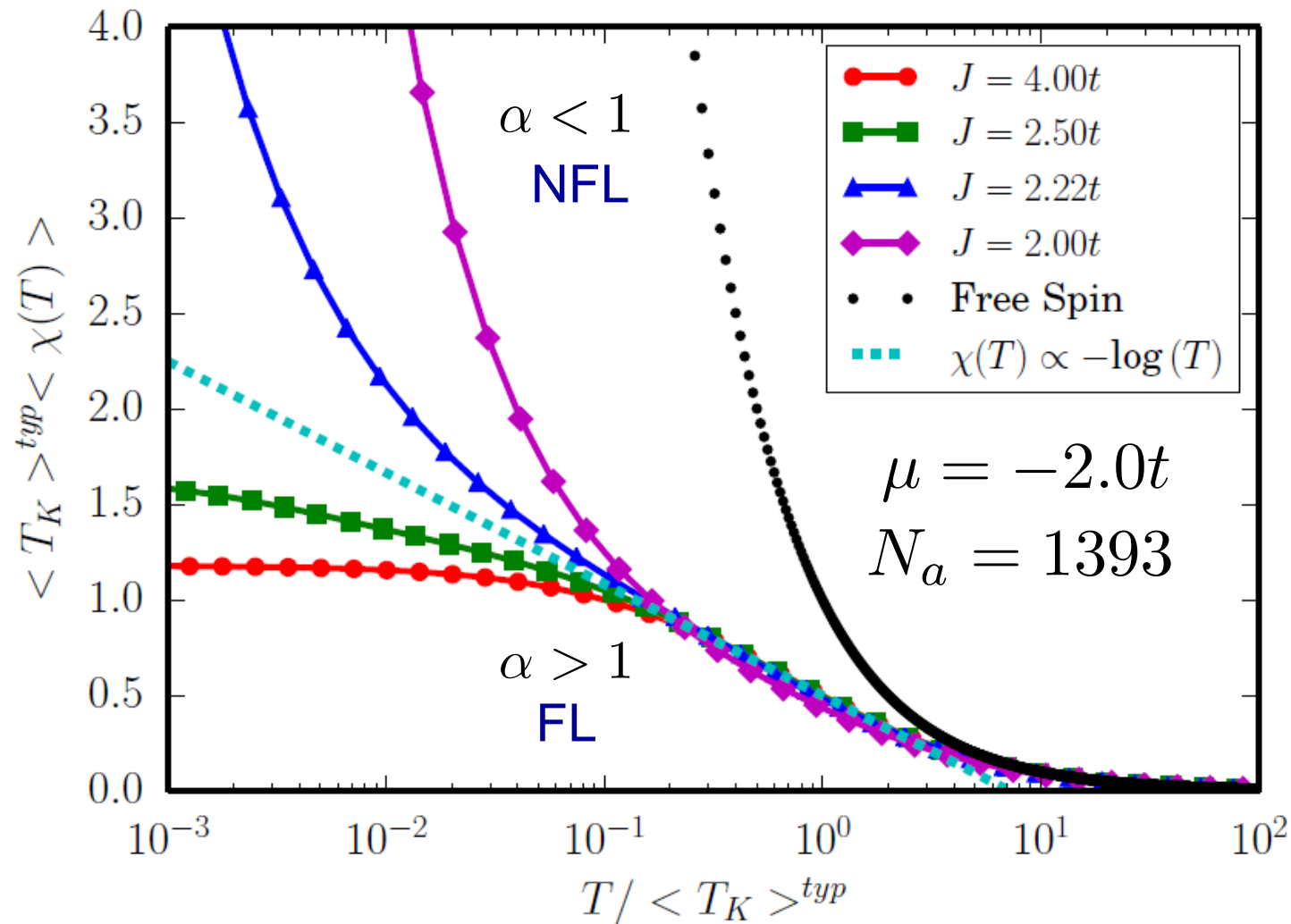
$$C/T = \gamma(T) \propto T^{\alpha-1}$$

$$\chi/\gamma \sim cte$$

$$1/T_1 T \propto T^{\alpha-2}$$

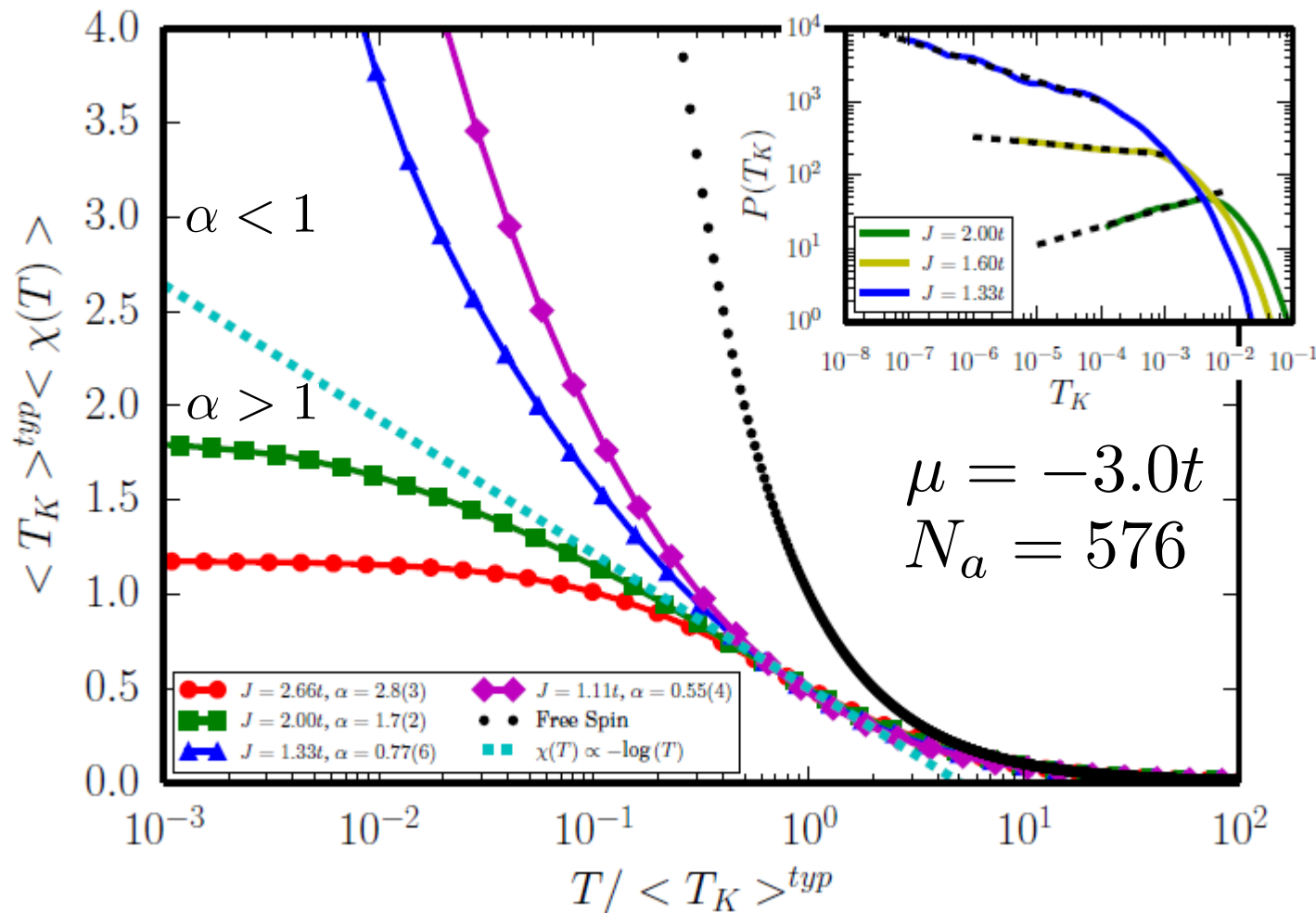
NFL behavior – Octagonal tiling

A power-law distribution of Kondo temperatures, leads to a power-law susceptibility

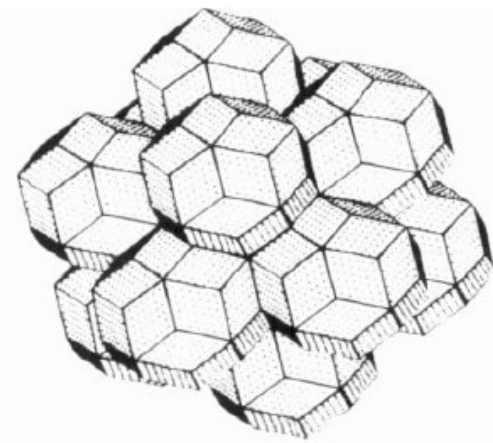


NFL behavior – Three dimensions

- Similar behavior in 3D: rhombohedra (Ammann-Kramer) tiling
- There seems to be little dimensionality dependence ($D = 2,3$)

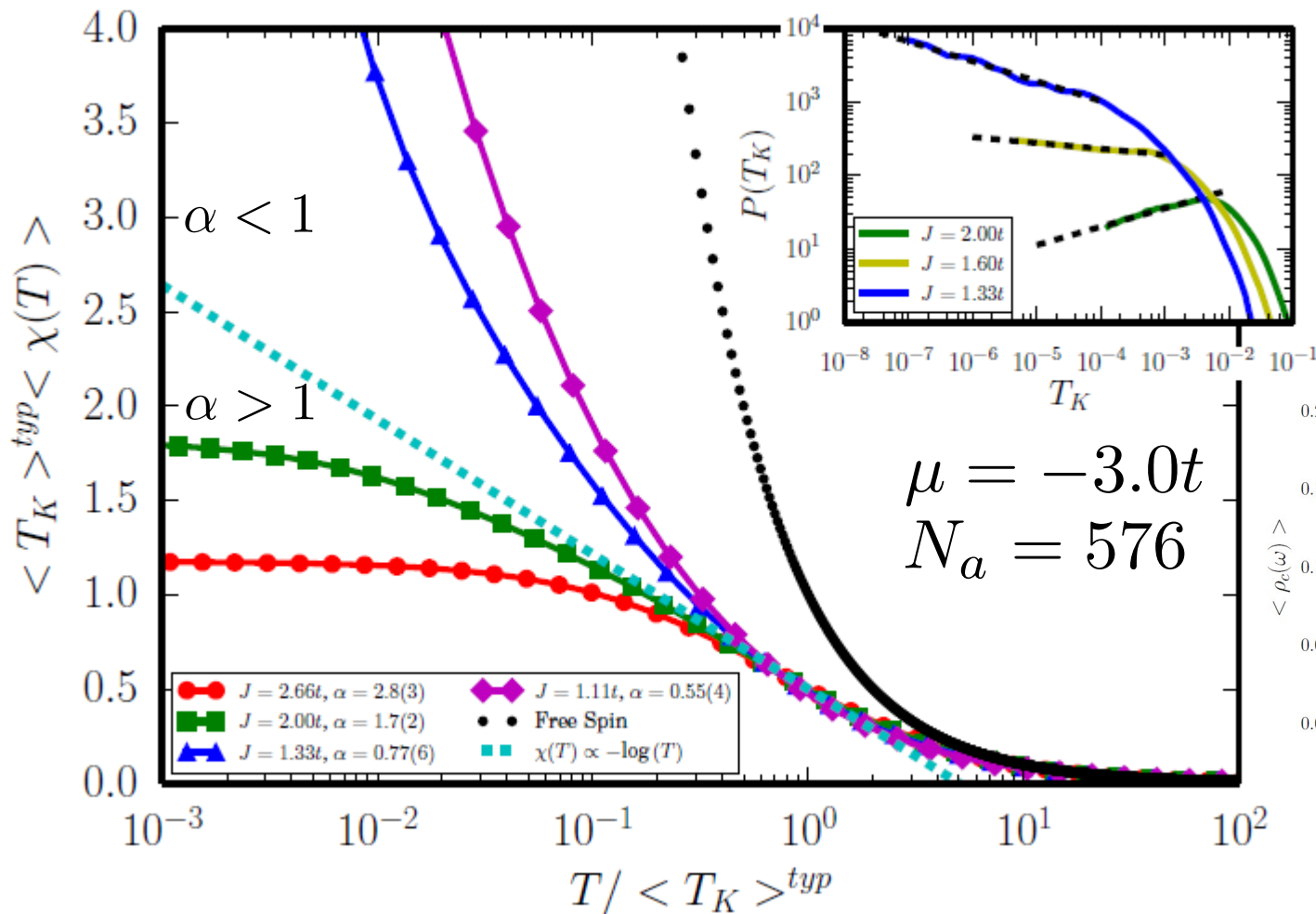


Provided by
M. Mihalkovic

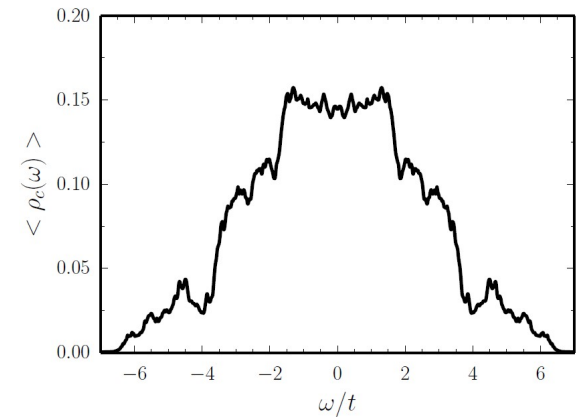


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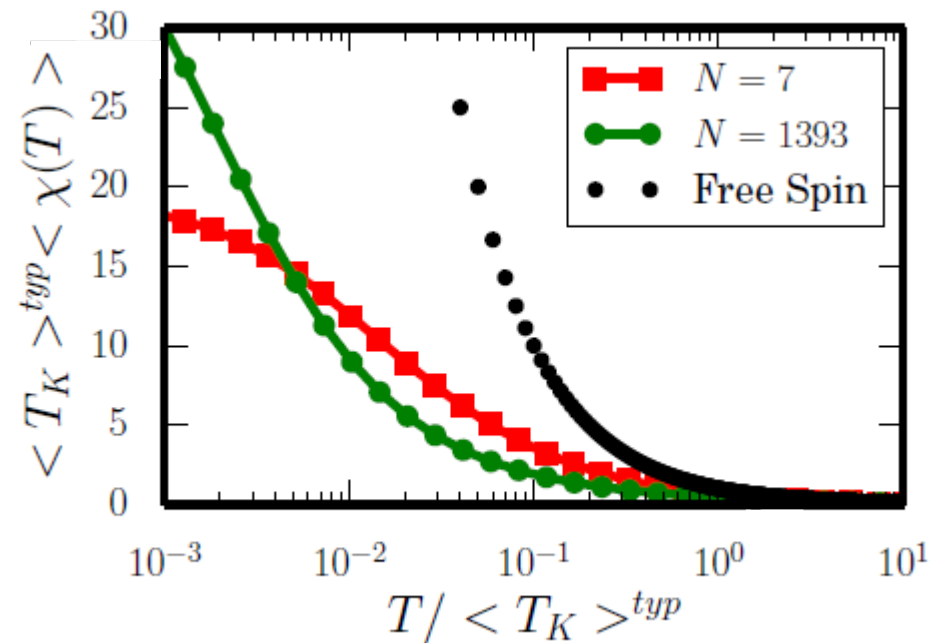
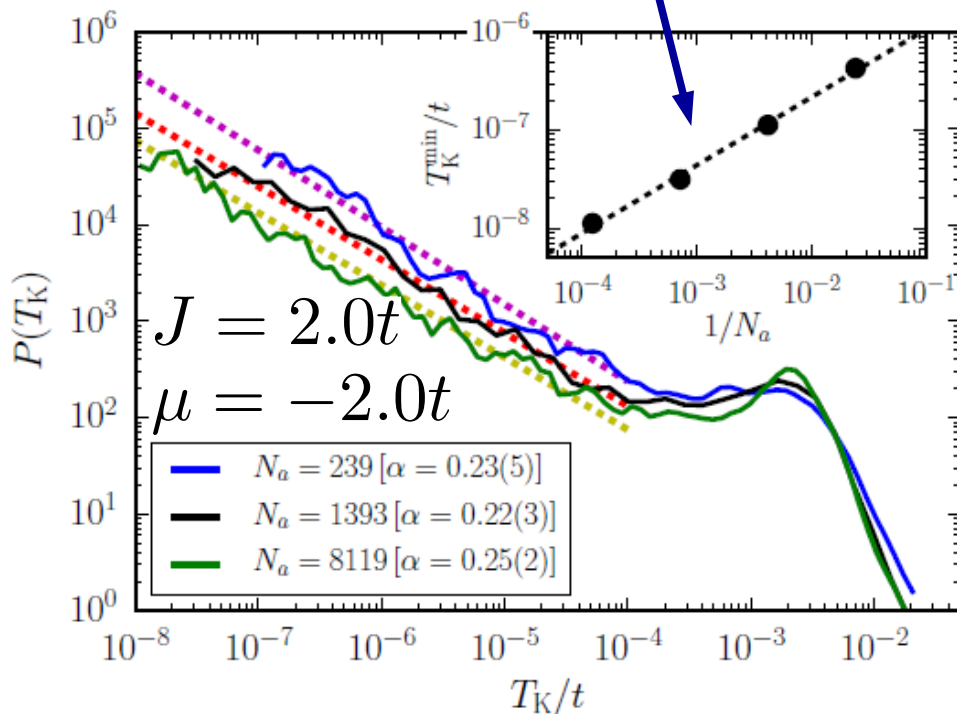
Provided by
M. Mihalkovic



FL behavior in small approximants

- For any approximant, there is a finite number of T_K 's, and therefore a **minimum one**.
- Below this T_K^{\min} , FL behavior is recovered.
- Only in an **infinite QC** there is true NFL behavior.

Minimum value of Kondo temperature: T_K^{\min}



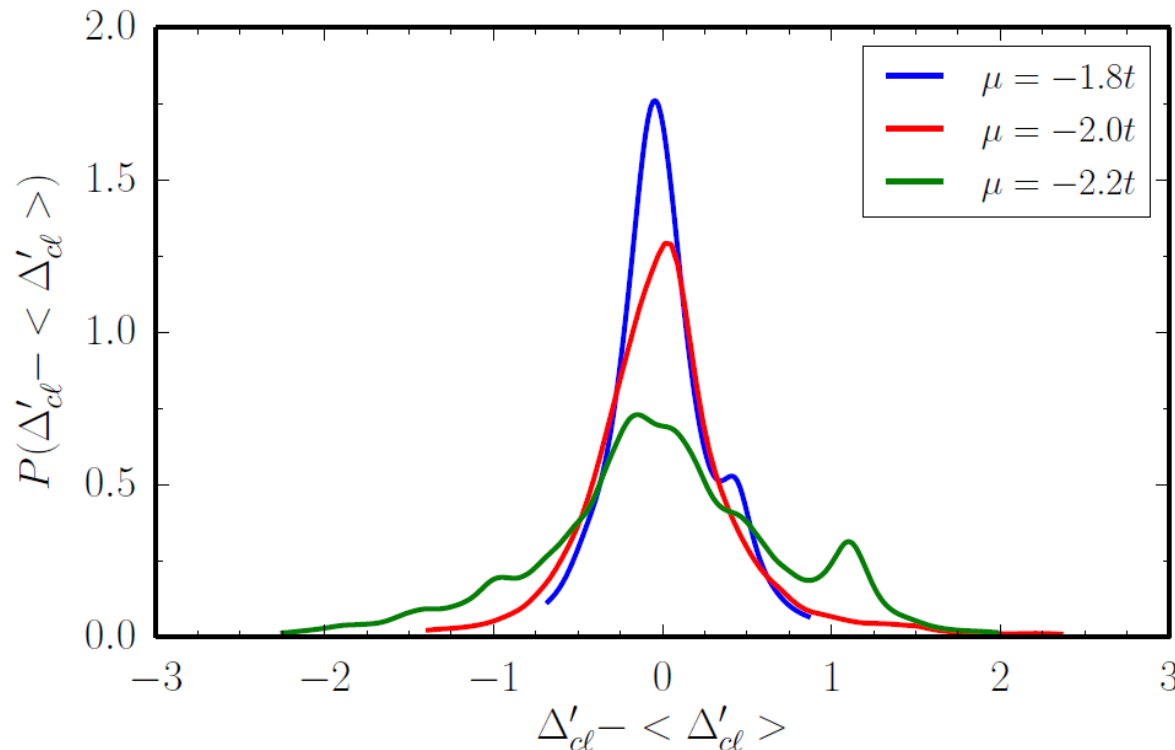
Why power-law? Metallic quasicrystal

- Define an “effective site energy” via the cavity function

$$G_{\ell\ell}^c(\omega) = 1/(\omega - \Delta_{cl}(\omega)) \quad \tilde{\varepsilon}_\ell = \Delta'_{cl}(0)$$

$$T_K^\ell = T_K^0 e^{-\tilde{\varepsilon}_\ell^2 / J \langle \rho_c(0) \rangle t^2} \quad P(\tilde{\varepsilon}) \sim e^{-\tilde{\varepsilon}^2 / 2\sigma^2} \quad P(T_K) \propto T_K^{\alpha-1}$$

$$\alpha = J \langle \rho_c(0) \rangle t^2 / 2\sigma^2$$



Gaussian-like tails in $P(\Delta'(0))$ immediately leads to singular behavior in $P(T_K)$

Akin to disordered metals

Tanasković/Miranda/Dobrosavljević
PRB **70**, 205108 (2004)

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- **Conclusions**

Conclusions

- Experiments: NFL behavior in the $\text{Au}_{51}\text{Al}_{34}\text{Yb}_{15}$ quasicrystal
 - No tuning of external parameters. Approximant different from the quasicrystal. Quenched by H but not by P
- Diluted Kondo impurities in metallic quasicrystals: Power-law distribution of T_K . Akin to *weakly* disordered systems
- Strong energy dependence. NFL behavior changes/disappears in small approximants (observed experimentally)
- Lattice problem: Diluted impurity scenario survives? (Most likely yes for thermodynamics!), Effects of inter-site spin correlations; Kondo coherence; Transport ...

Thank you very much for
your attention!

Phys. Rev. Lett. **115**, 036403 (2015)

