A MICROWAVE REALIZATION OF PENROSE TILING

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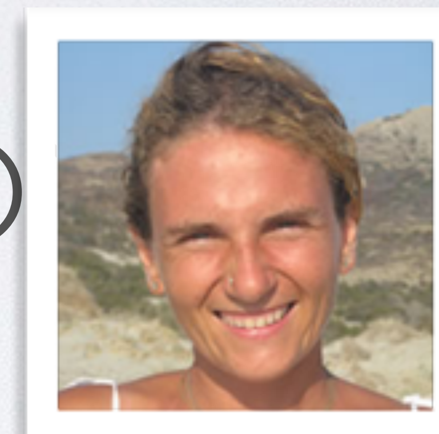
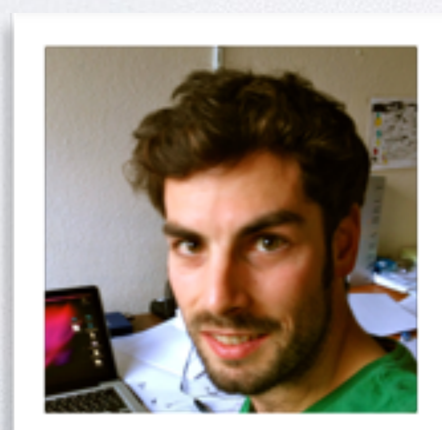
CROWAVE REALIZATIO
PENROSE TILING



Ulrich Kuhl

Patrizia Vignolo



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Matthieu Bellec

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J.-M. Gambaudo

SOMETHING WRONG ?

10:55 - 11:15	Coffee break
11:15 - 13:00	**Electrons on tilings - Mortessagne, Kalouguine, Akkermans- chair: S. Aubry**
11:15 - 11:50	› Gap labeling and energy landscape in a 2D quasicrystal: A microwave experiment - *Fabri*
11:50 - 12:25	› An ansatz for single electron wavefunctions in quasicrystalline potentials - *Pavel Kalougu*
12:25 - 13:00	› Topological properties of Fibonacci quasicrystals : A scattering analysis of Chern number
13:00 - 14:30	Lunch

QUANTUM/CLASSICAL WAVES

free particle

$$-\Delta\psi(\vec{r}) = E\psi(\vec{r})$$

$(\hbar^2/2m = 1)$

microwave cavity

$$-\Delta\psi(\vec{r}) = k^2\psi(\vec{r})$$

$$\psi = E_z, B_z$$

+ varying potential

$$\left[-\Delta + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

+ varying permittivity

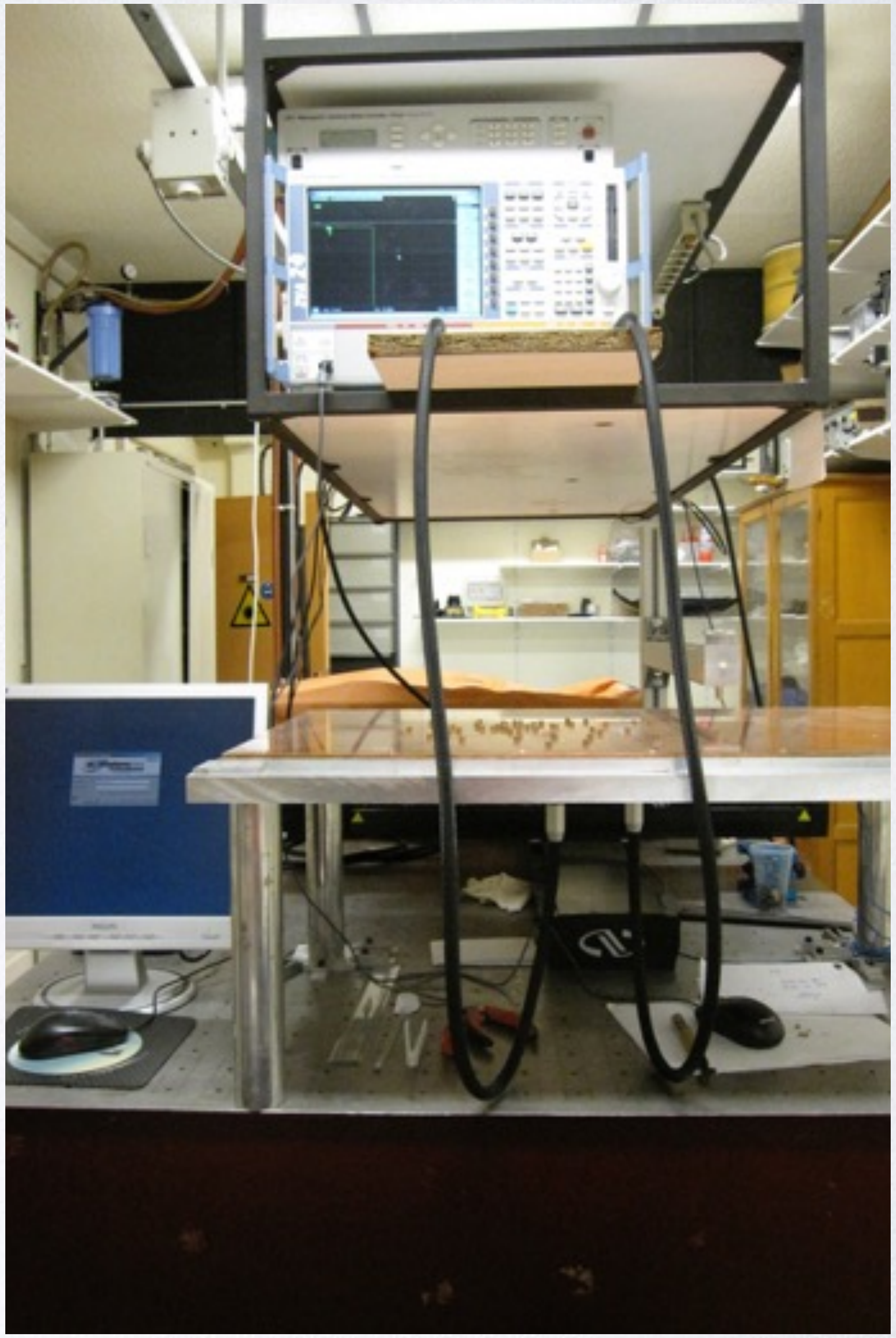
$$\left[-\Delta + \big(1-\varepsilon(\vec{r})\big)k^2\right]\psi(\vec{r}) = k^2\psi(\vec{r})$$

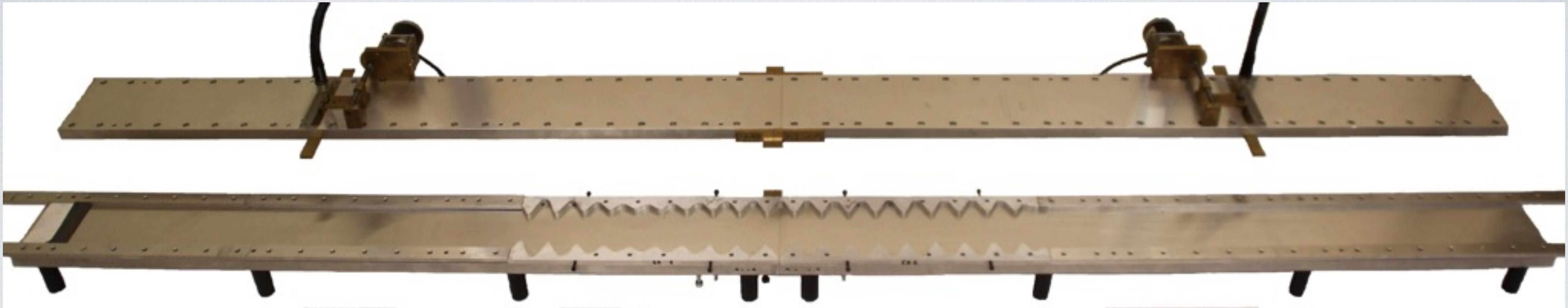
quantum chaos and mesoscopic physics with microwaves

See H.-J. Stöckmann's book (quantum chaos); Akkermans & Montambaux's book (mesoscopic physics)

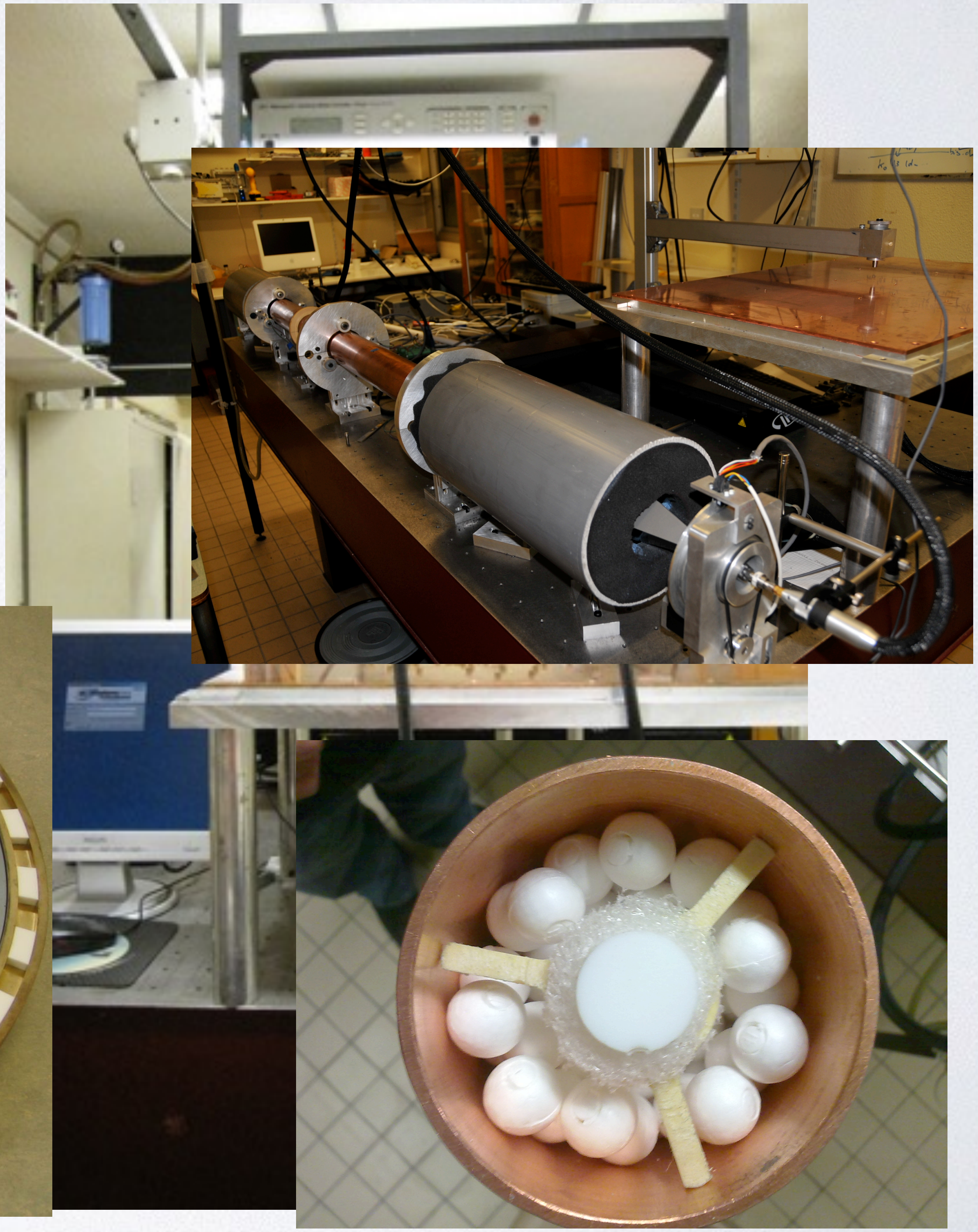
EXPERIMENTAL SETUP

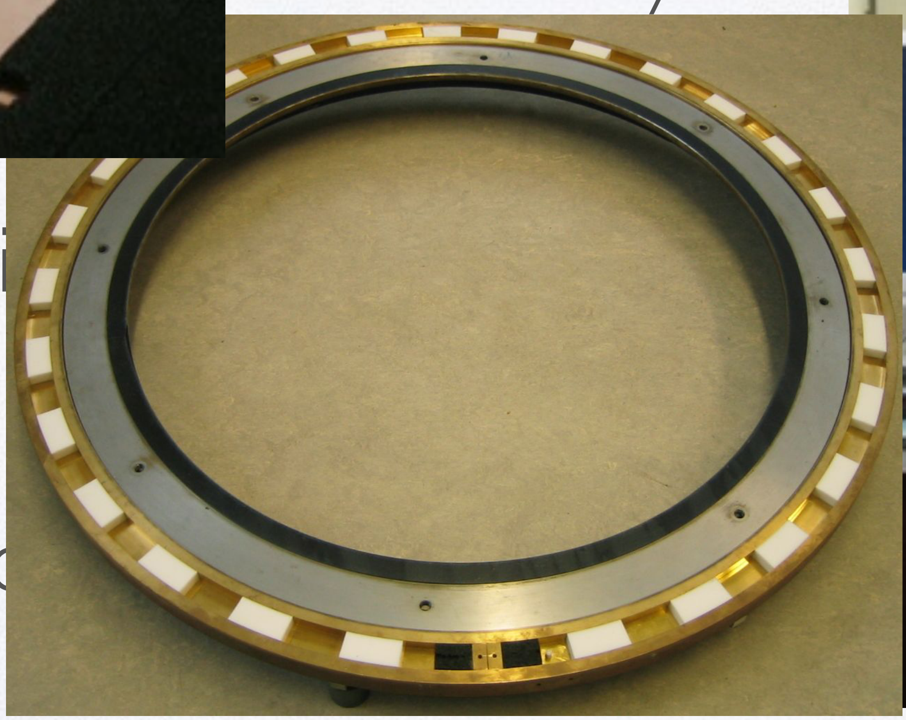
- network analyzer provides the scattering S-matrix (~10GHz)
- (quasi-)1D/2D/3D geometries
- closed or partially open boundary conditions
- TM or TE polarization
- chaotic cavities, periodic or disordered lattices





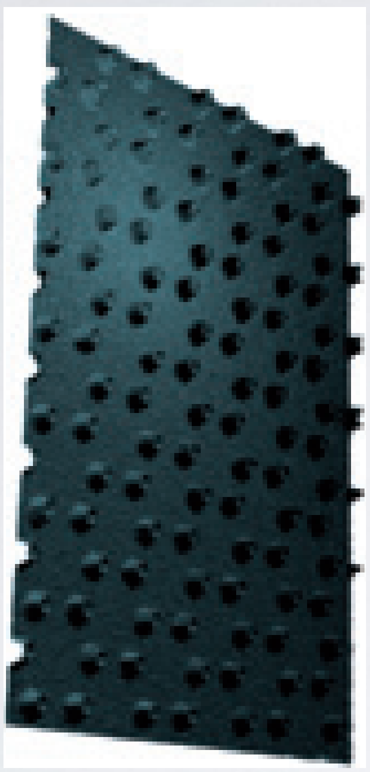
- network analyzer provides the
  10GHz)
  ometries
  en boundary



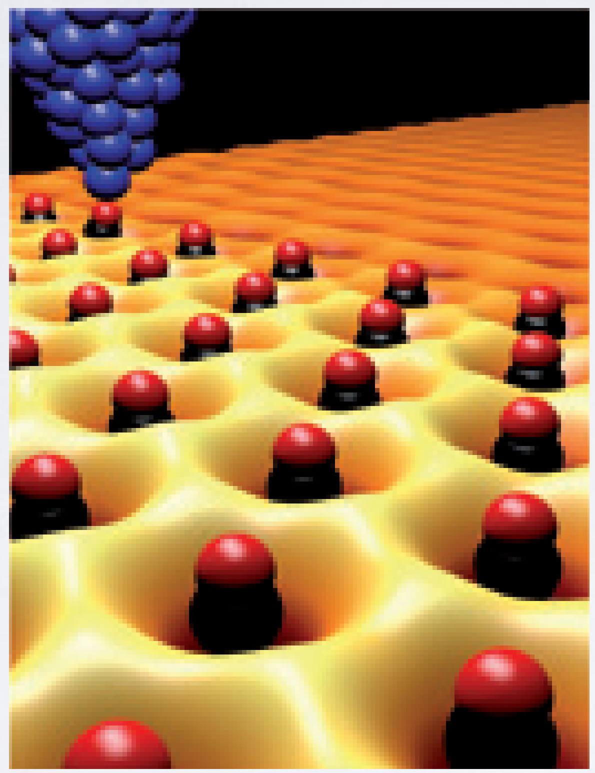


- TM or TE polari
- chaotic cavities,
  disordered lattic

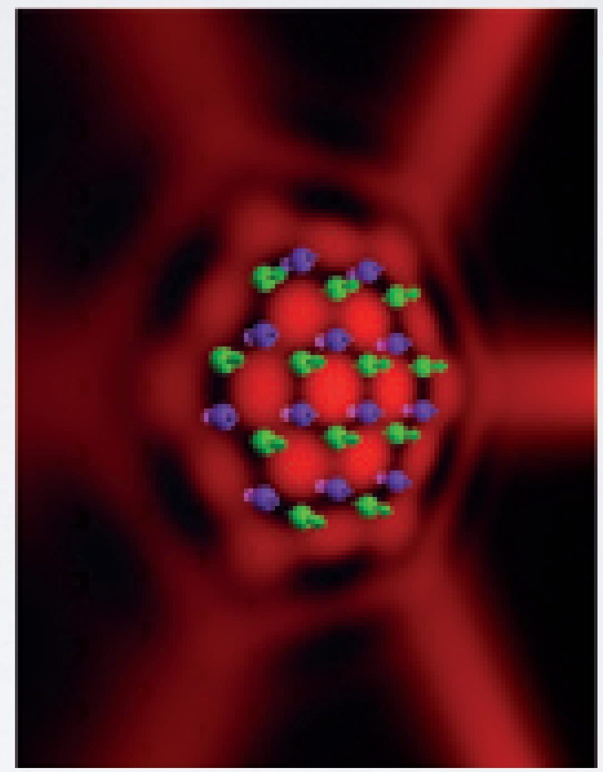
ARTIFICIAL DIRAC & TOPOLOGICAL MATERIALS



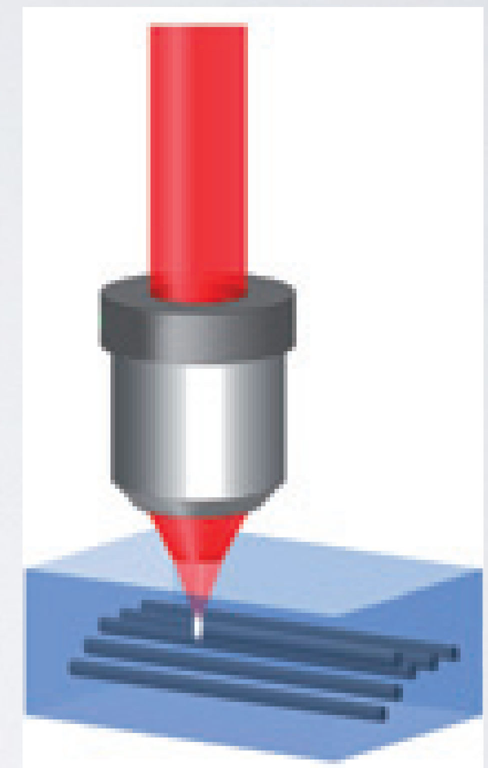
2DEG
nanopatterned SC



2DEG
molecular lattice



cold atoms
optical lattice



light/microwave
refractive index lattice

0D Polaritons
micropillar lattice

nature nanotechnology
REVIEW ARTICLE
PUBLISHED ONLINE: 4 SEPTEMBER 2013 | DOI: 10.1038/NNANO.2013.161

Artificial honeycomb lattices for electrons, atoms and photons

Marco Polini[1]*, Francisco Guinea[2], Maciej Lewenstein[3,4], Hari C. Manoharan[5,6] and Vittorio Pellegrini[1,7]

OUTLINE

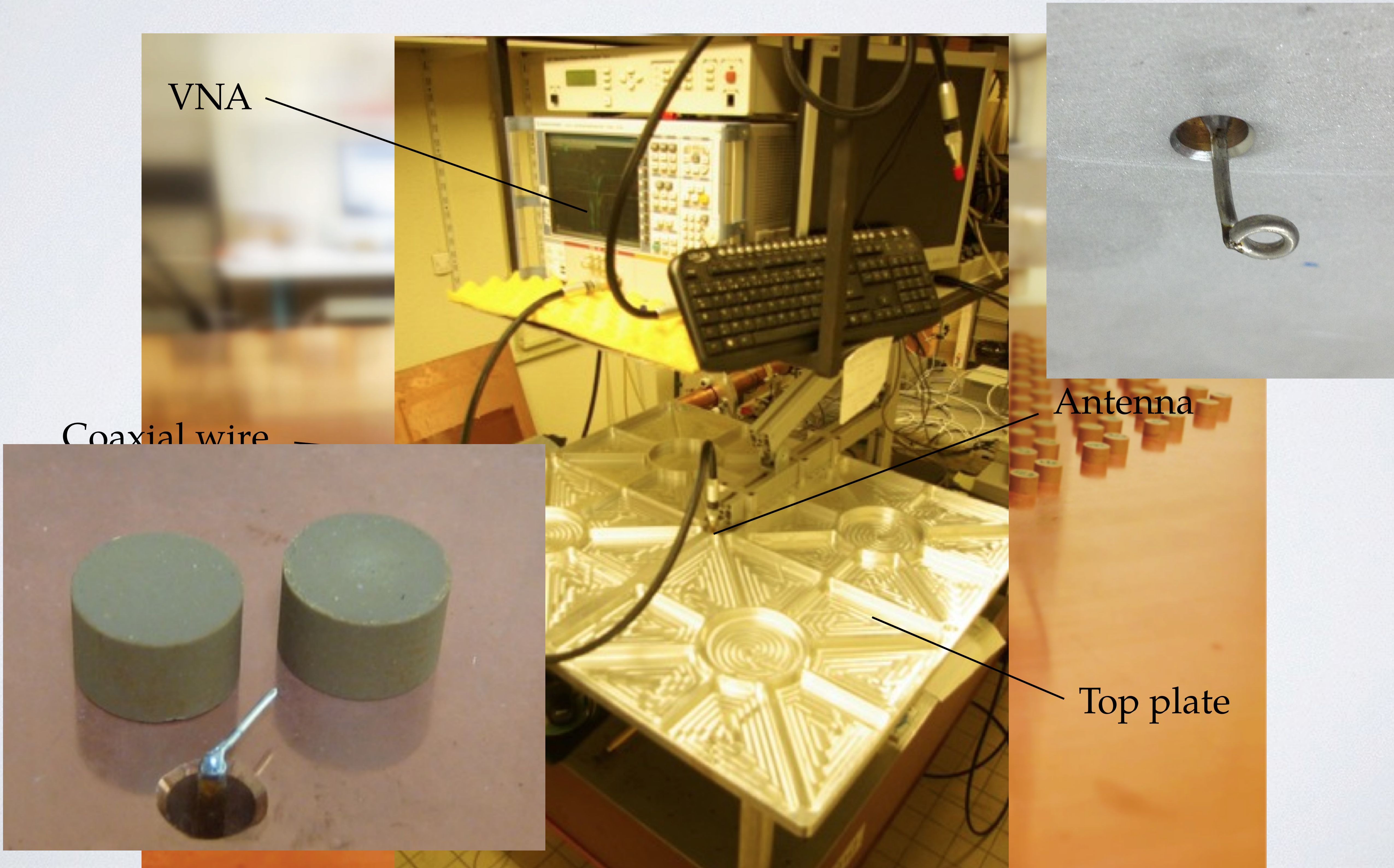
1. From microwave dielectric resonator to tight-binding lattices

   dielectric resonator, TE mode, evanescent coupling, LDOS & eigenstates

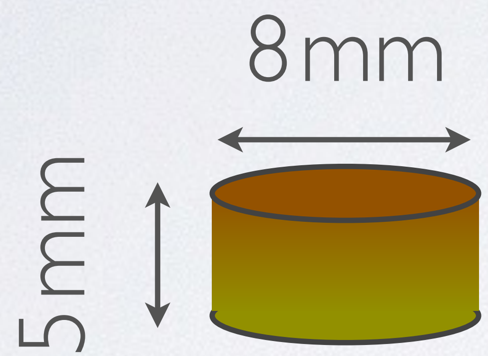
2. Experimental realization of a Penrose tiling quasicrystal

   irregular staircase IDOS, dominant coupling, energy landscape, gap labeling

NICE MICROWAVE SET-UP



MICROWAVE RESONATOR



Dielectric ceramic (ZrSnTiO):

- high permittivity $\varepsilon = 37$
- low loss $Q \simeq 7000$

Depending on the excitation:

TM modes: $\psi(\vec{r}) = E_z(\vec{r})$ energy everywhere

(particle with positive energy)

TE modes: $\psi(\vec{r}) = B_z(\vec{r})$ energy essentially inside

(particle with negative energy)

ISOLATED RESONATOR

reflection measurement:

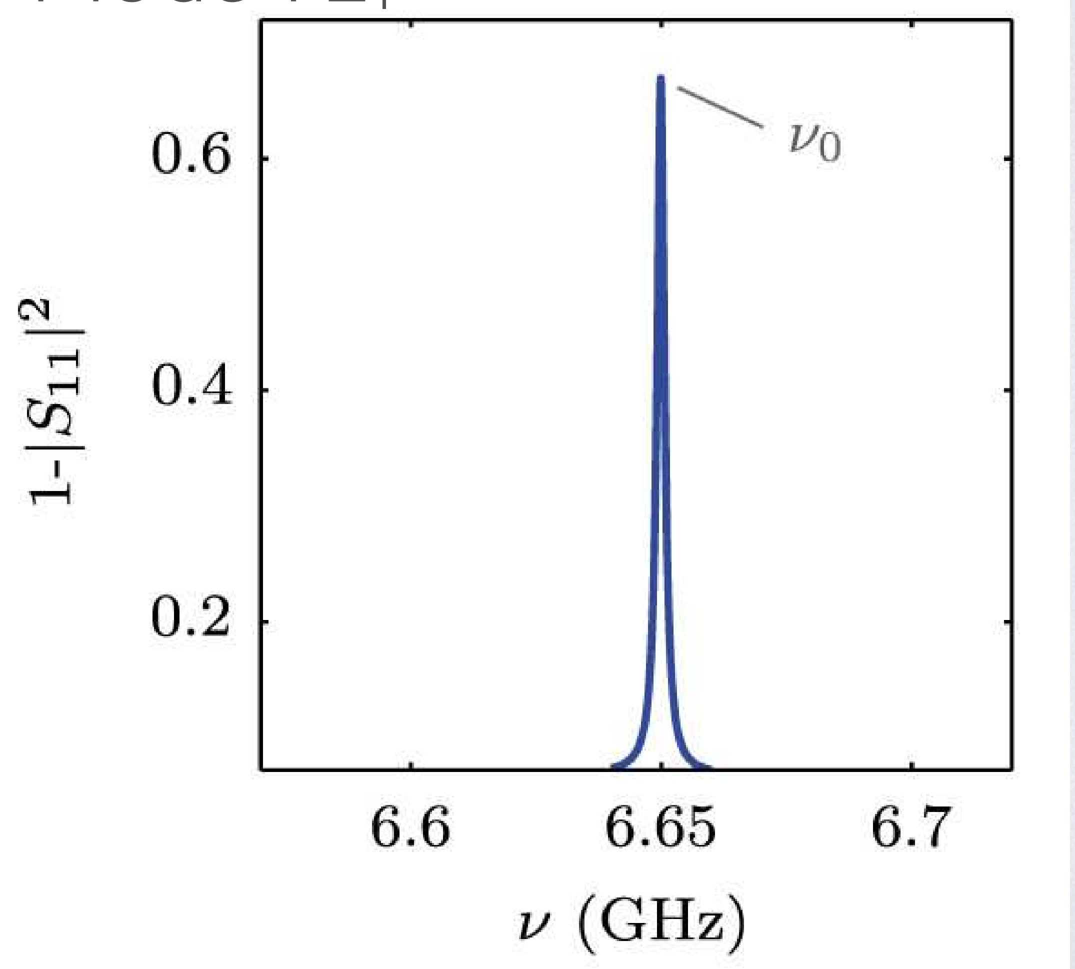
$$S_{11}(\nu) = 1 - \mathrm{i}\sigma \frac{|\Psi_0(\mathbf{r}_1)|^2}{\nu - \nu_0 + \mathrm{i}\Gamma}$$

σ : antenna coupling (constant and weak)

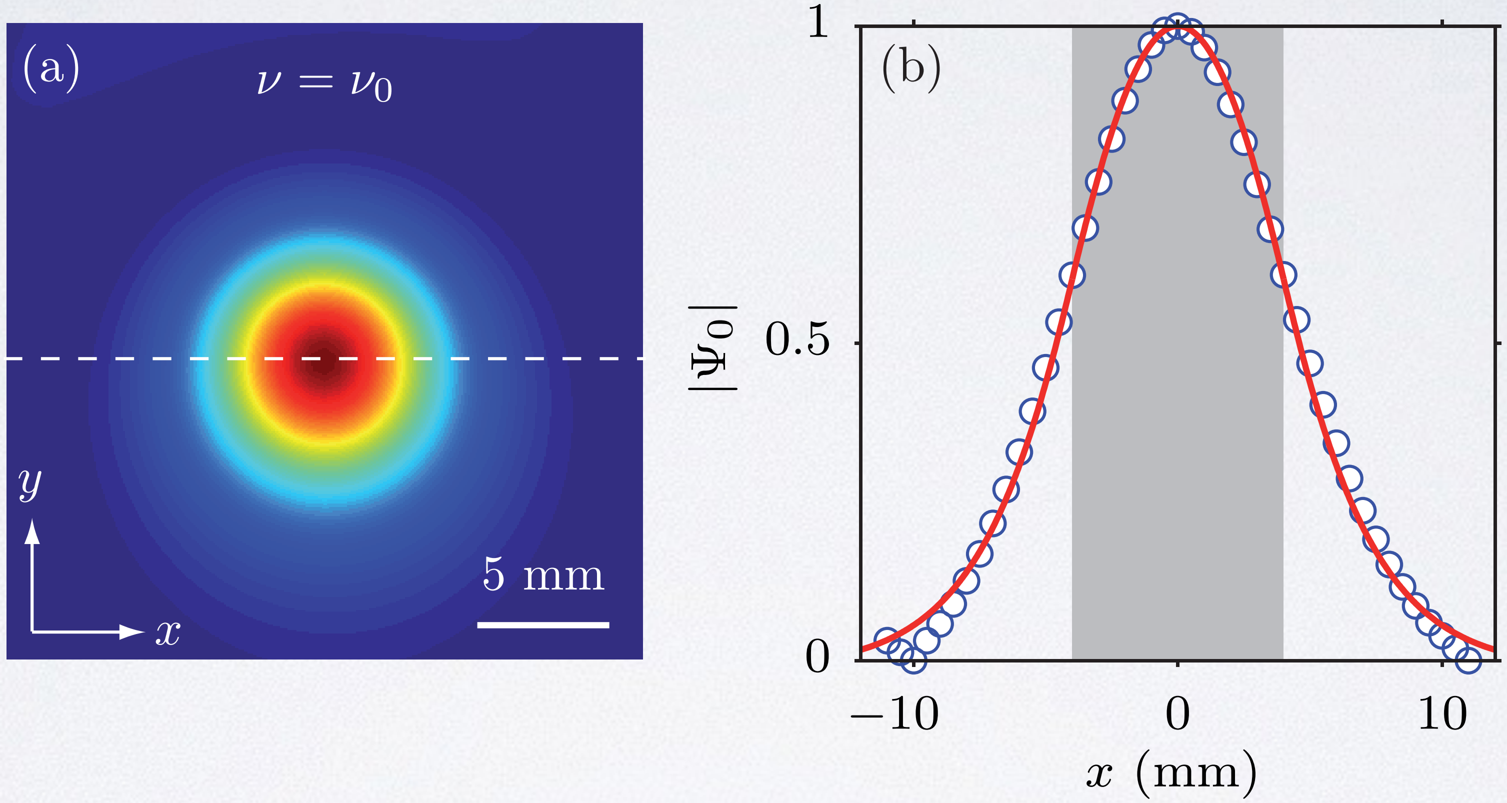
Γ^{-1} : lifetime $(\Gamma \simeq 2\,\mathrm{MHz})$

$$\hookrightarrow \; 1 - |S_{11}(\nu_0)|^2 \simeq \frac{2\sigma}{\Gamma} |\Psi_0(\mathbf{r}_1)|^2$$

Mode TE1



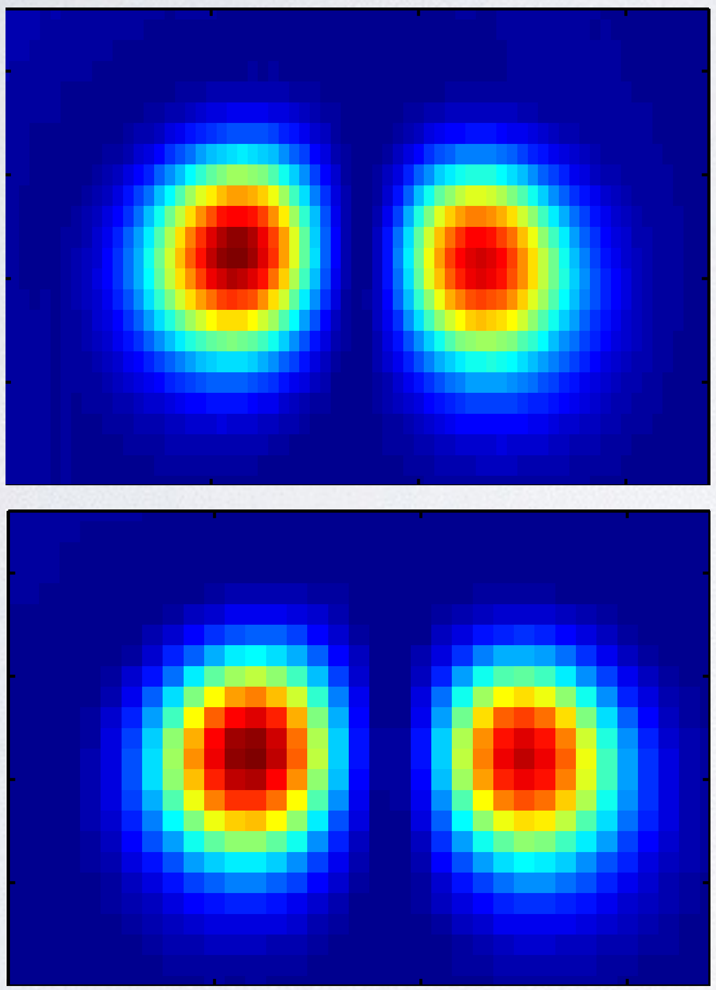
"BESSEL ORBITAL"

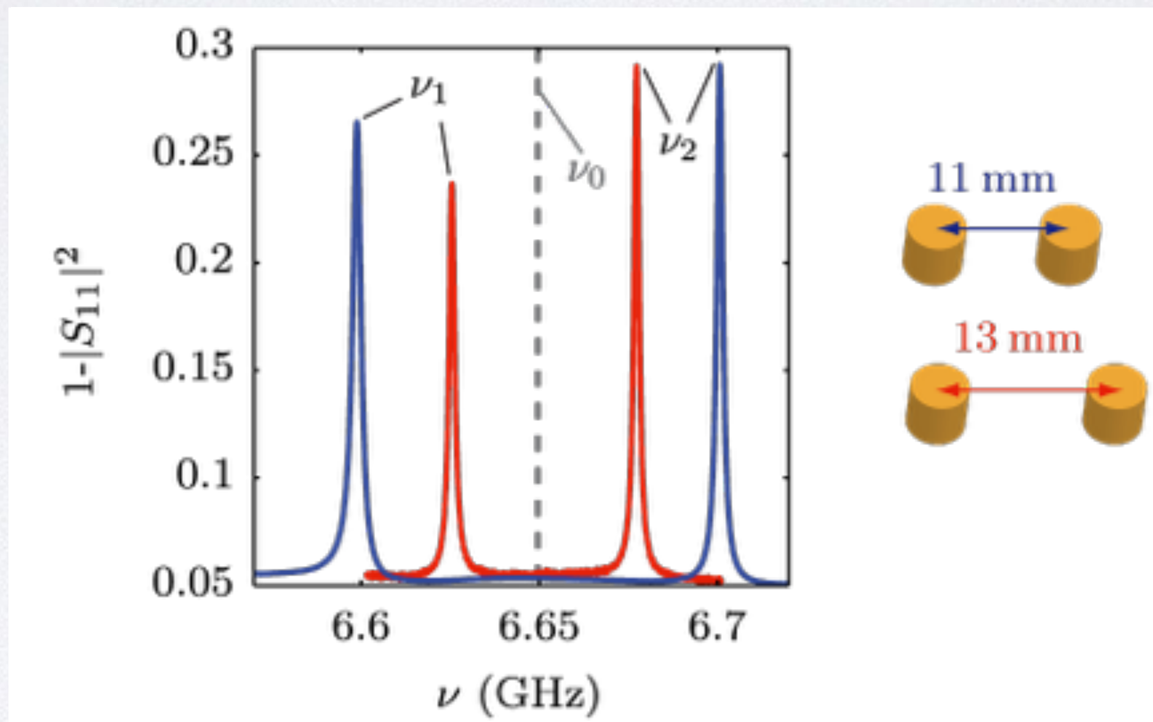


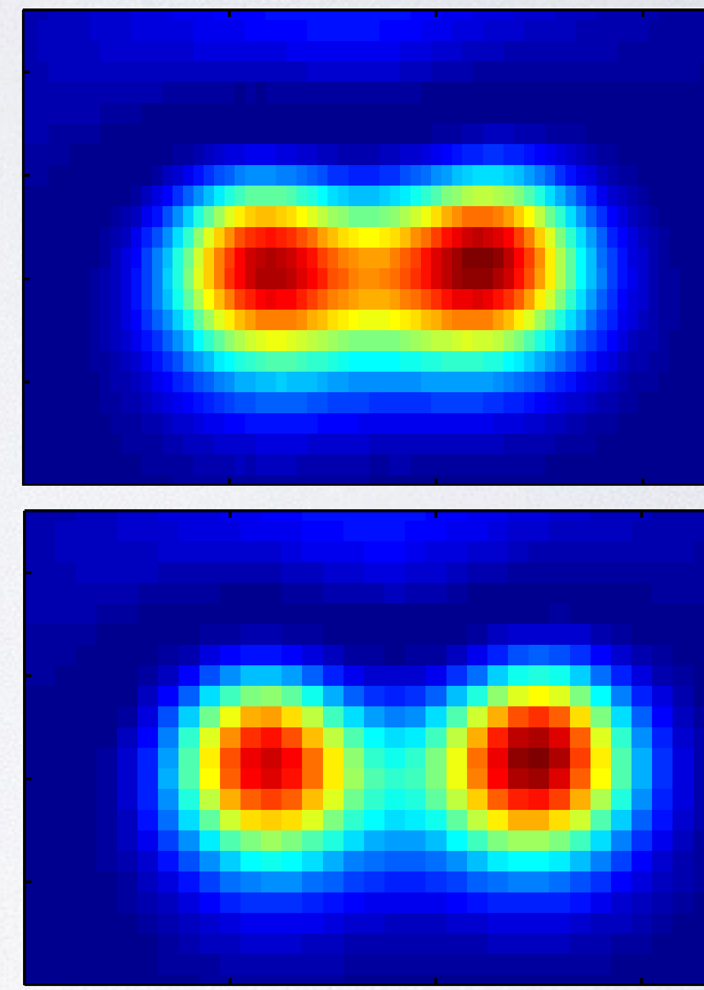
$$B_z(r,z) = B_0 \sin\left(\frac{\pi}{h} z\right) \times \begin{cases} J_0(\gamma_j r) & \text{if } r < r_D, \\ \alpha K_0(\gamma_k r) & \text{if } r > r_D, \end{cases}$$

$$\gamma_j = \sqrt{\left(\frac{2\pi\nu_0 n}{c}\right)^2 - \left(\frac{\pi}{h}\right)^2} \qquad \gamma_k = \sqrt{\left(\frac{\pi}{h}\right)^2 - \left(\frac{2\pi\nu_0}{c}\right)^2}$$

TIGHT-BINDING COUPLING

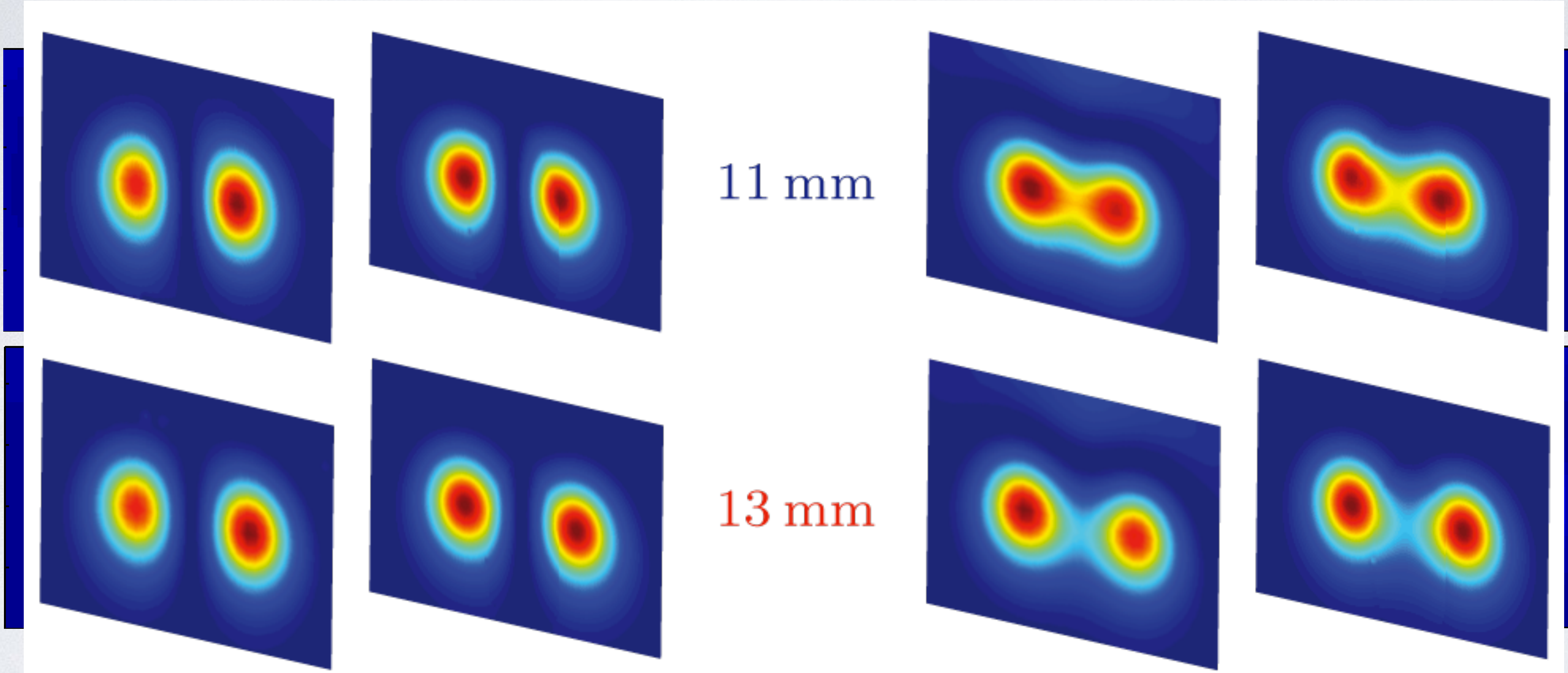






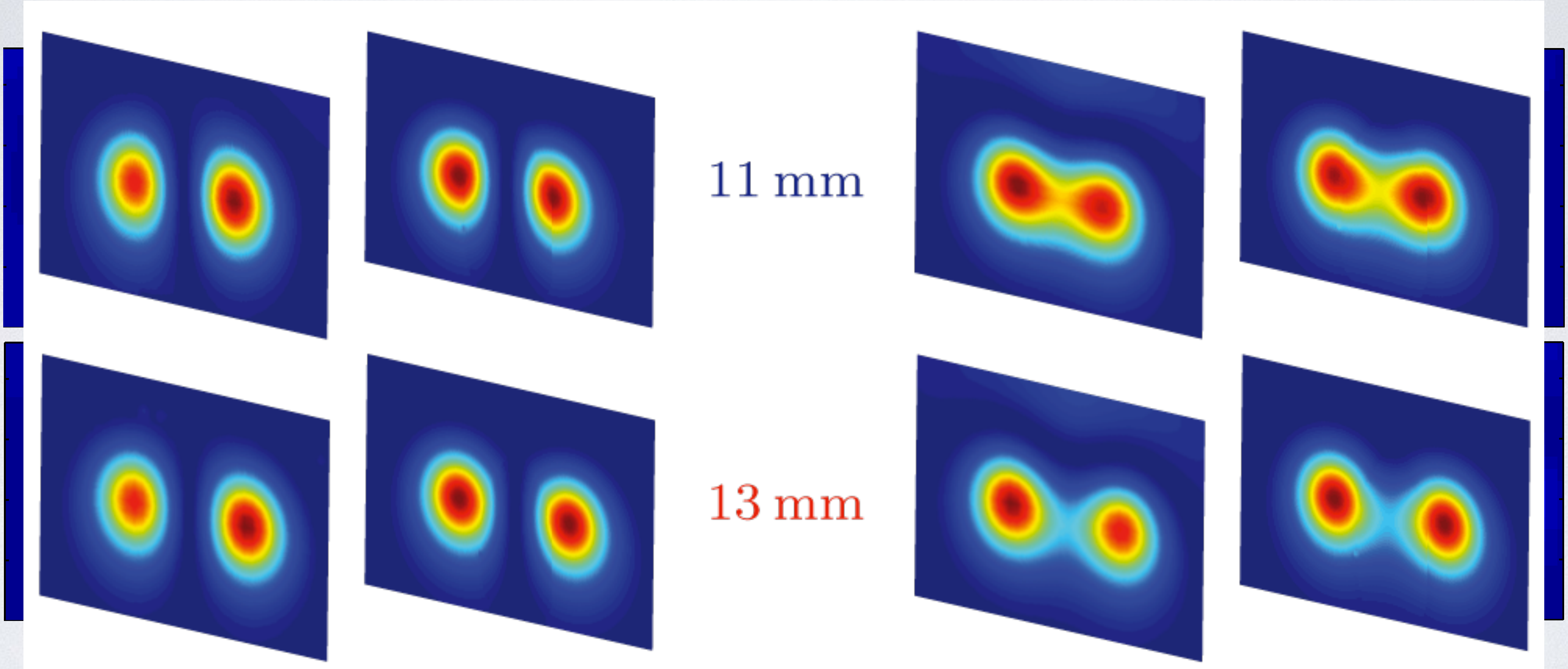
- symmetric splitting of the eigenfrequencies

TIGHT-BINDING COUPLING



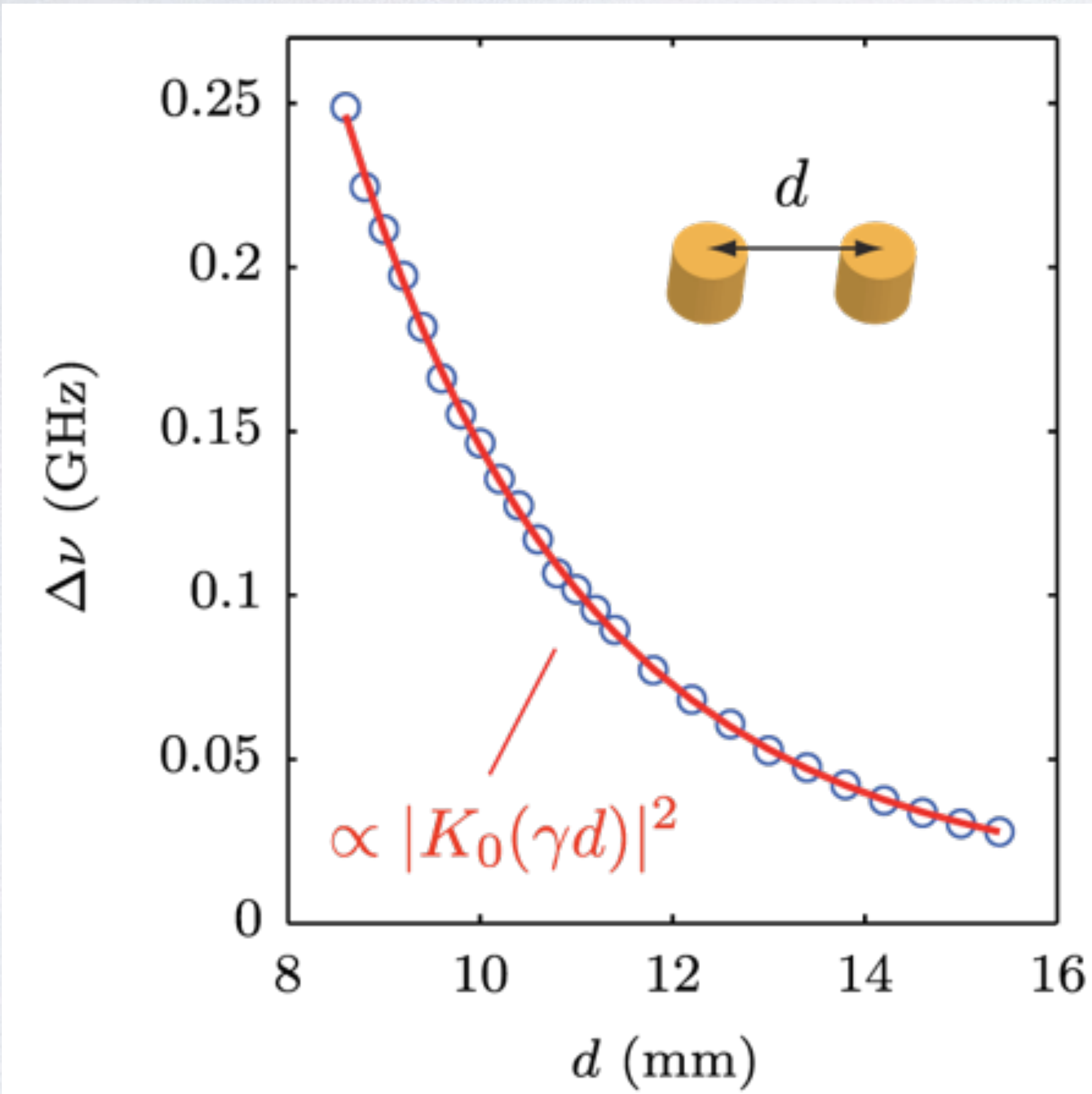
- symmetric splitting of the eigenfrequencies
- linear superposition of isolated eigenfunctions
- antisymmetric state associated with the lowest frequency

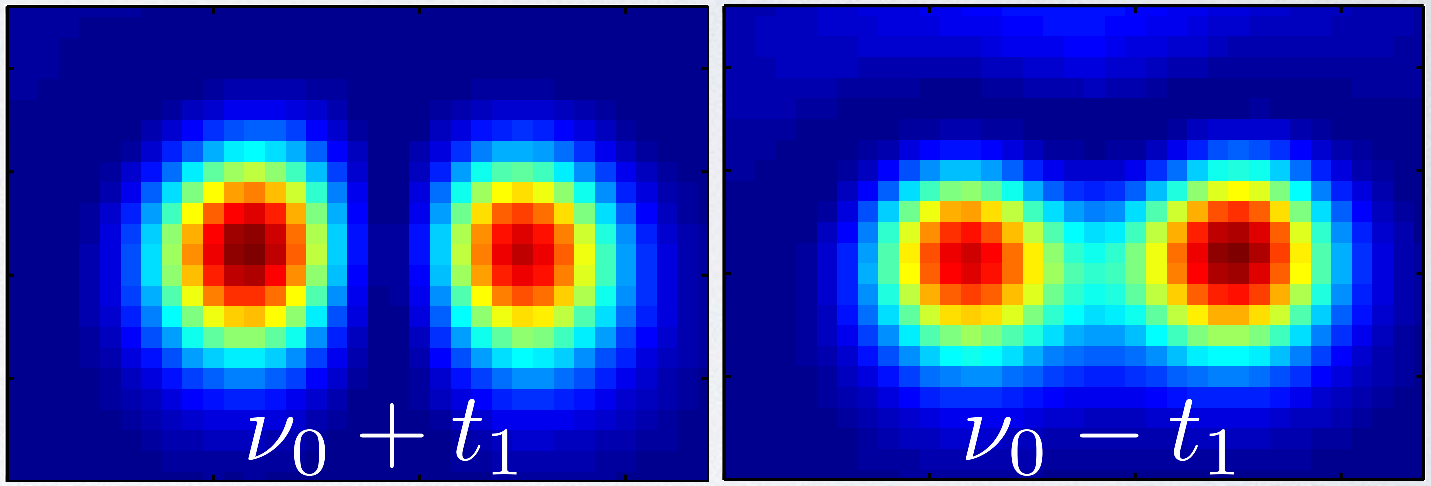
TIGHT-BINDING COUPLING



$$H_{\mathrm{TB}} = \begin{pmatrix} \nu_0 & -t_1 \\ -t_1 & \nu_0 \end{pmatrix} \quad t_1 < 0$$

TIGHT-BINDING COUPLING





$$H(d) = \begin{pmatrix} \nu_0 & -t_1(d) \\ -t_1(d) & \nu_0 \end{pmatrix}$$

splitting: $\Delta\nu(d) = 2|t_1(d)|$

$$|t_1(d)| = \alpha\,|K_0(\gamma d/2)|^2 + \delta$$

$\alpha \simeq 1.95\,\mathrm{GHz}, \gamma \simeq 0.32\,\mathrm{mm}^{-1}, \delta \simeq 0.005\,\mathrm{GHz}$

Similar values of the fitting parameters are obtained for benzene-like, square and honeycomb lattices.

PRB **88**, 115437 (2013)

DATA PROCESSING

A direct access to the density of states and intensity of the eigenstates through:

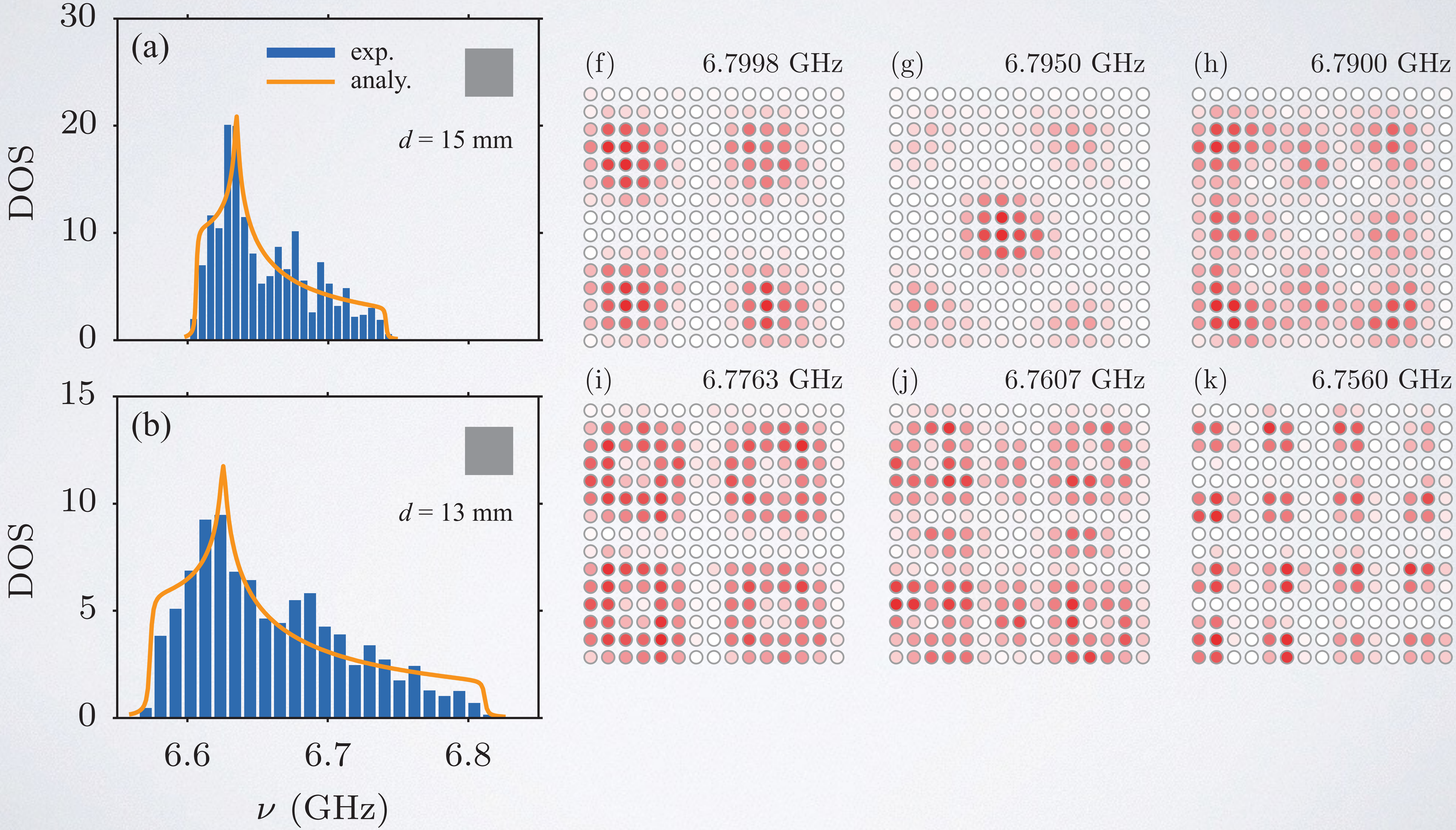
$$g(\mathbf{r}_1,\nu) = \frac{|S_{11}(\nu)|^2}{\langle |S_{11}|^2\rangle_\nu}\varphi'_{11}(\nu) \qquad \arg\left[S_{11}(\nu)\right] = \varphi_{11}(\nu)$$

$$g(\mathbf{r}_1,\nu) = -\frac{\sigma}{\Gamma\left\langle |S_{11}|^2\right\rangle_\nu}\boxed{\sum_n |\Psi_n(\mathbf{r}_1)|^2\delta(\nu-\nu_n)}$$

local density of states, DOS by averaging

for a given eigenfrequency: measure the local intensity

SQUARE LATTICE



OUTLINE

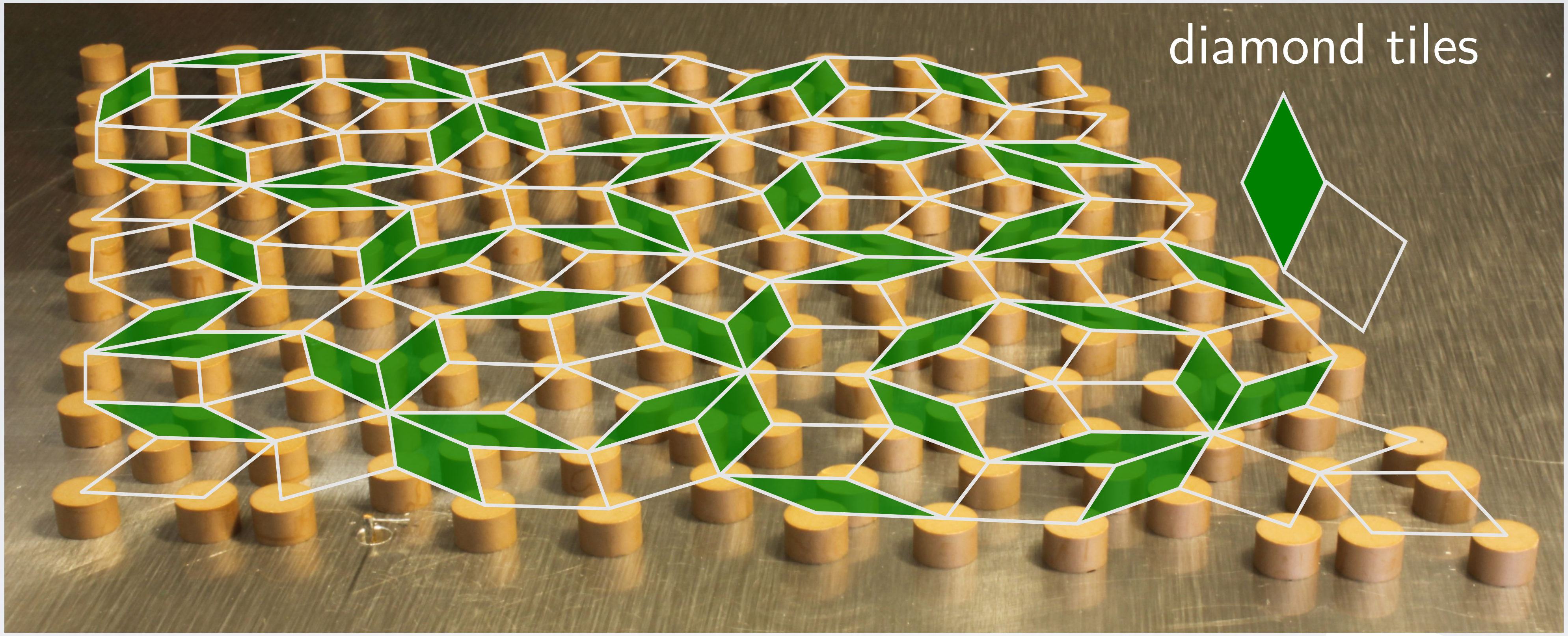
1. From microwave dielectric resonator to tight-binding lattices

   dielectric resonator, TE mode, evanescent coupling, LDOS & eigenstates

2. Experimental realization of a Penrose tiling quasicrystal

   irregular staircase IDOS, dominant coupling, energy landscape, gap labeling

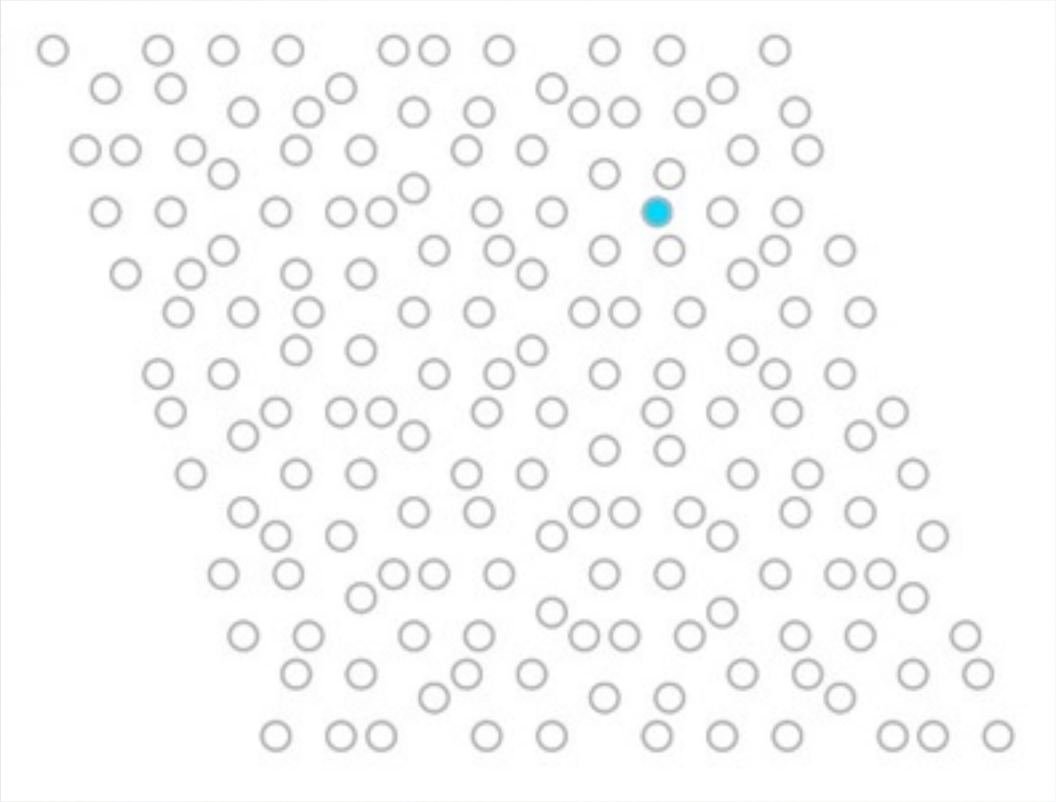
MICROWAVE PENROSE TILING

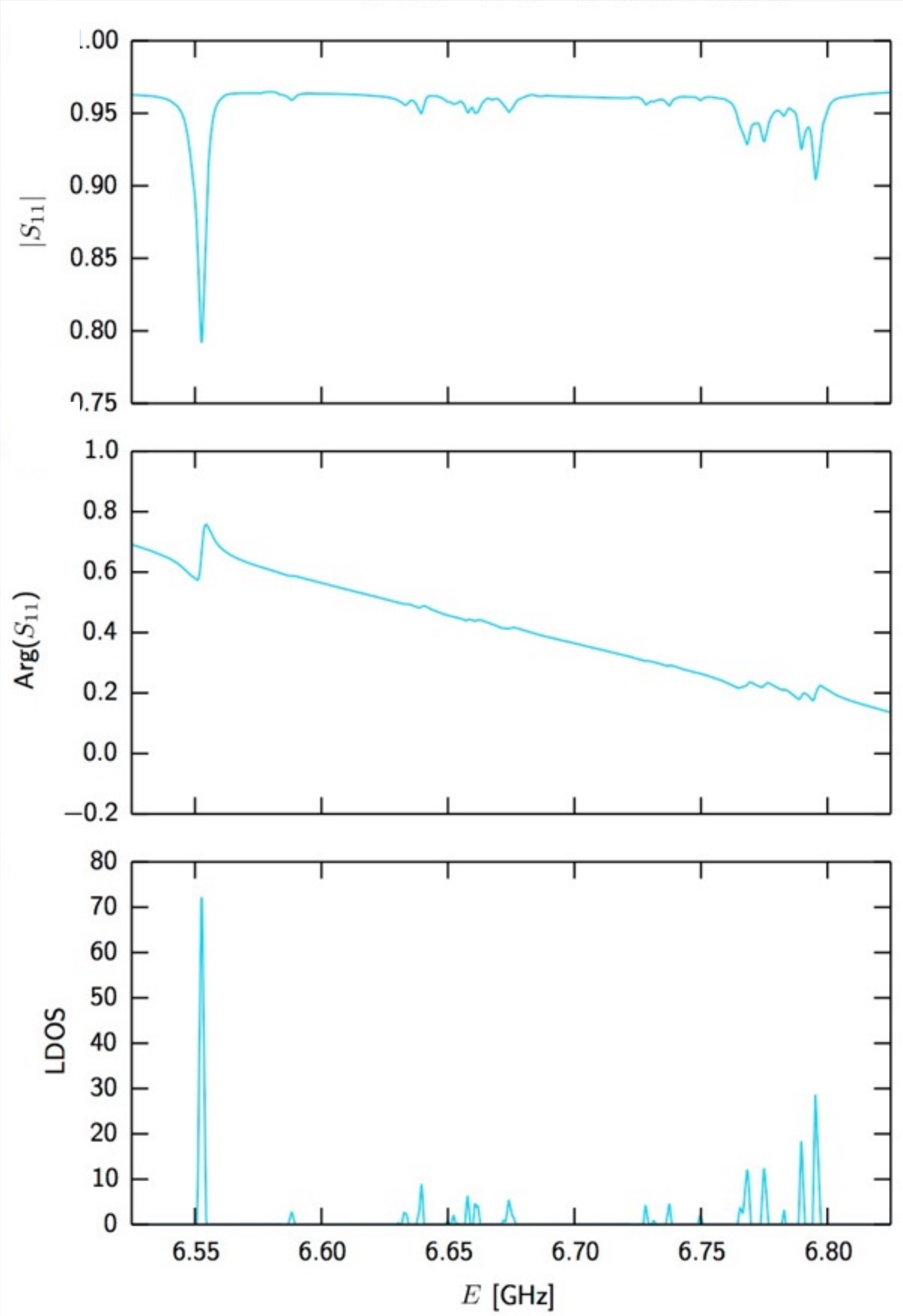


- Penrose lattice with diamond tiles
- 164 resonators placed at each diamond vertex

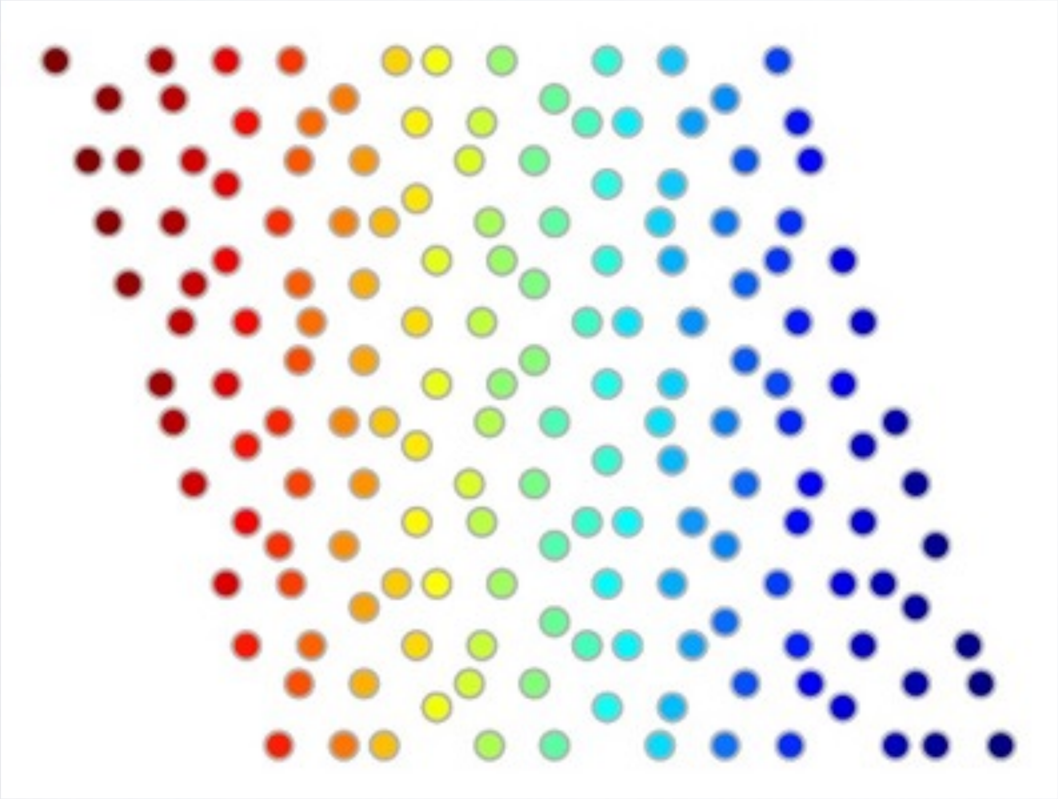
$$H = E_b \sum_i |i\rangle\langle i| + \sum_{i,j,i\neq j} t_{ij} |i\rangle\langle j|$$

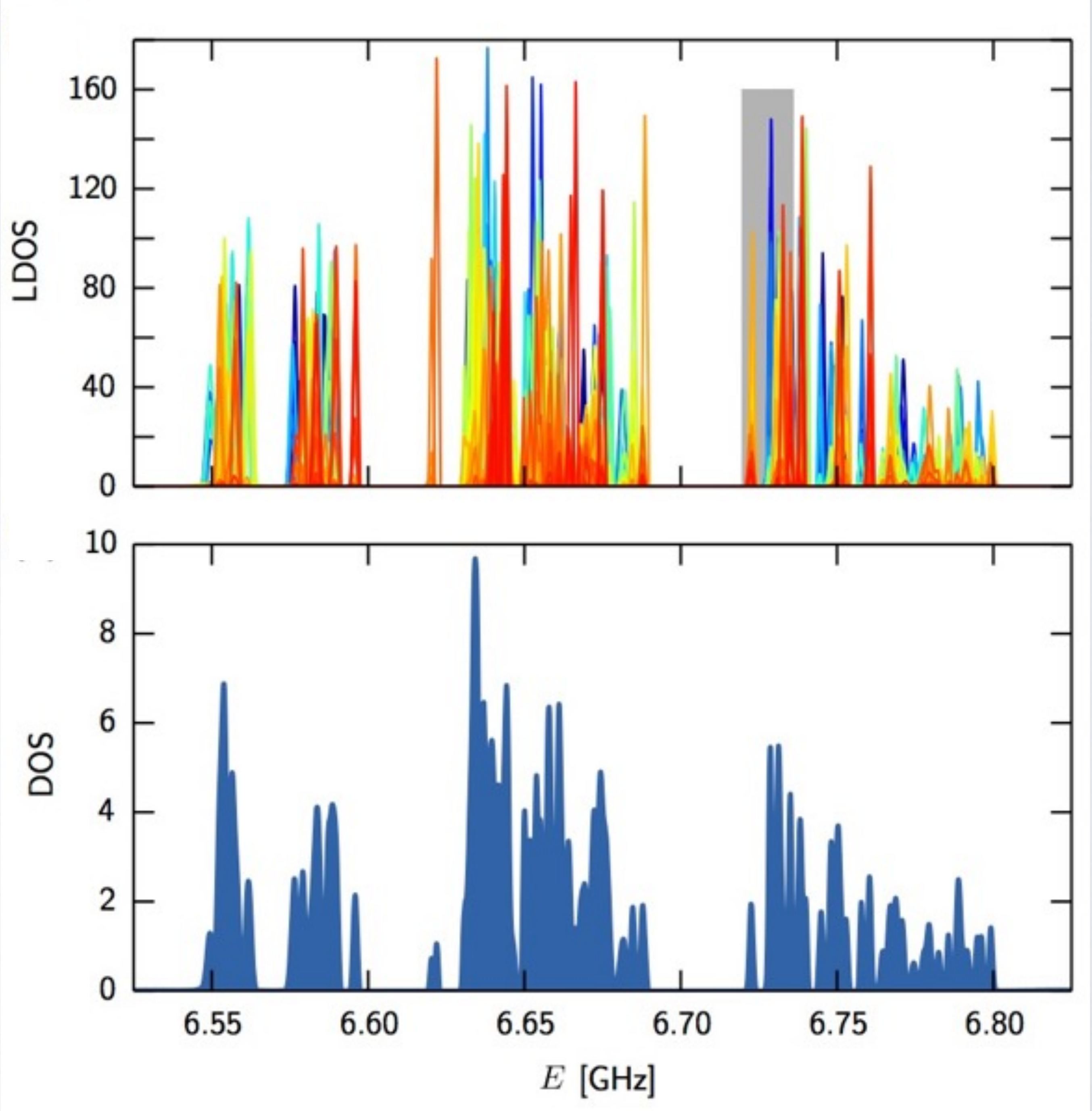
(LOCAL) DENSITY OF STATES



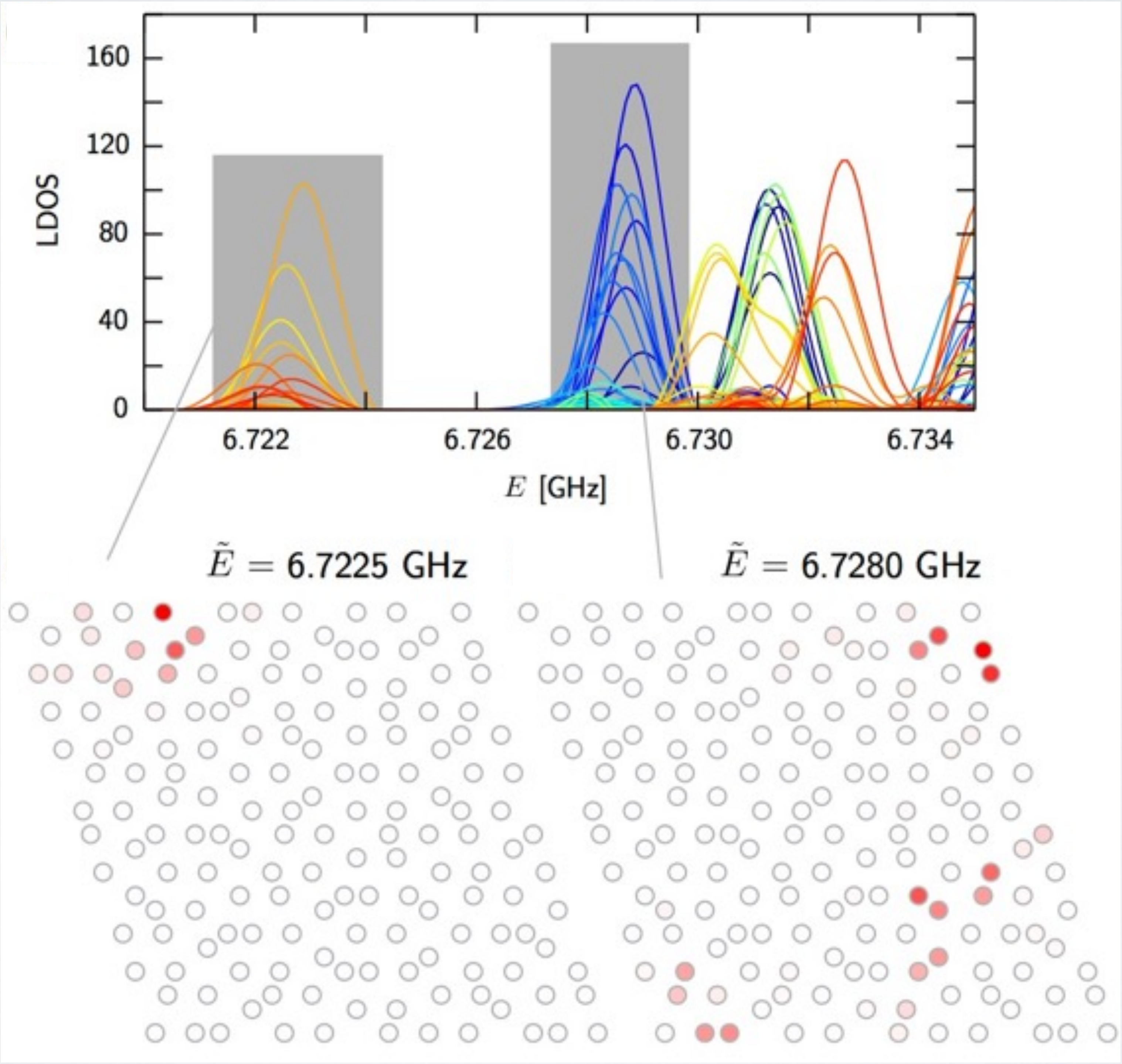


(LOCAL) DENSITY OF STATES

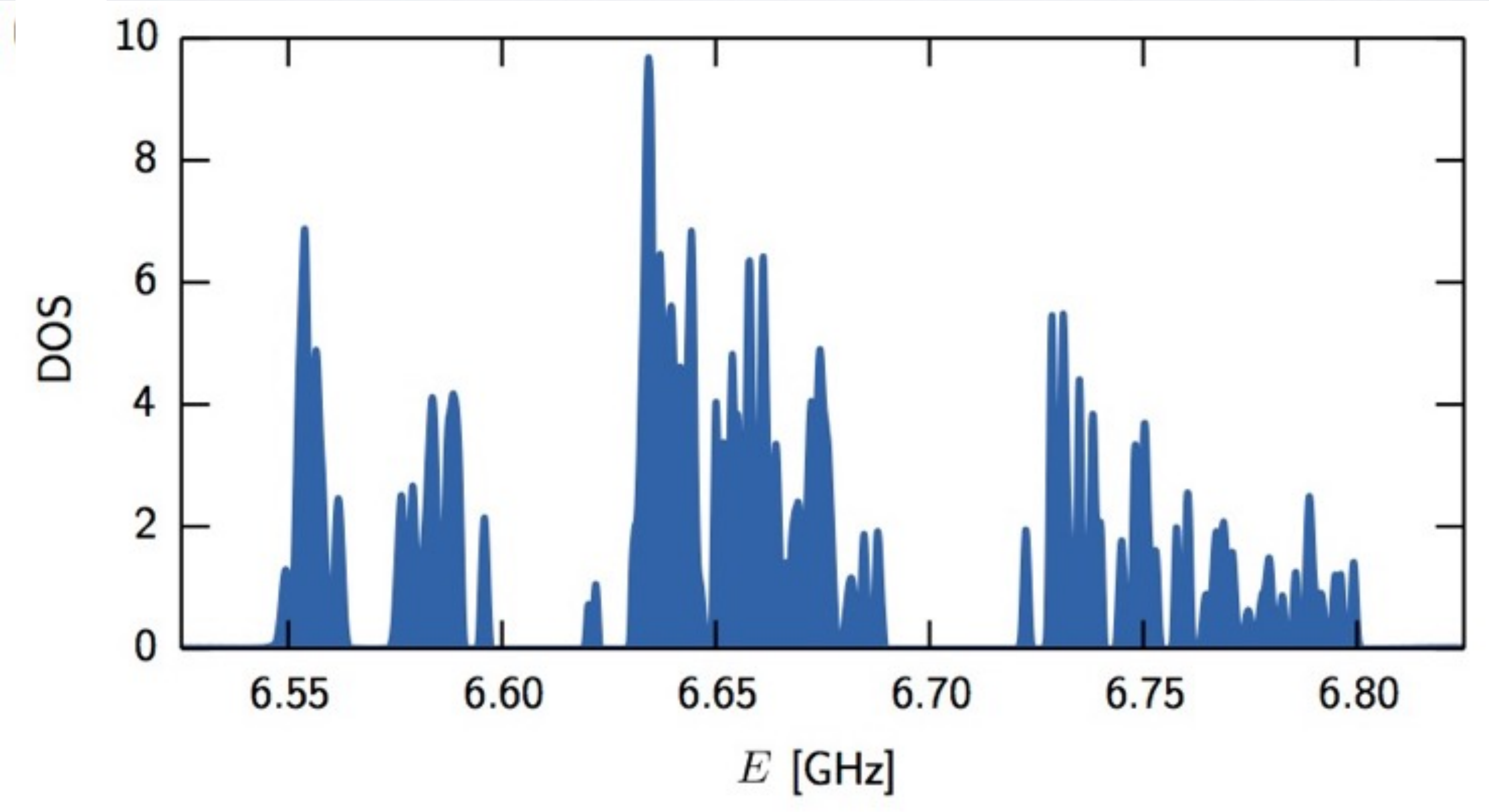




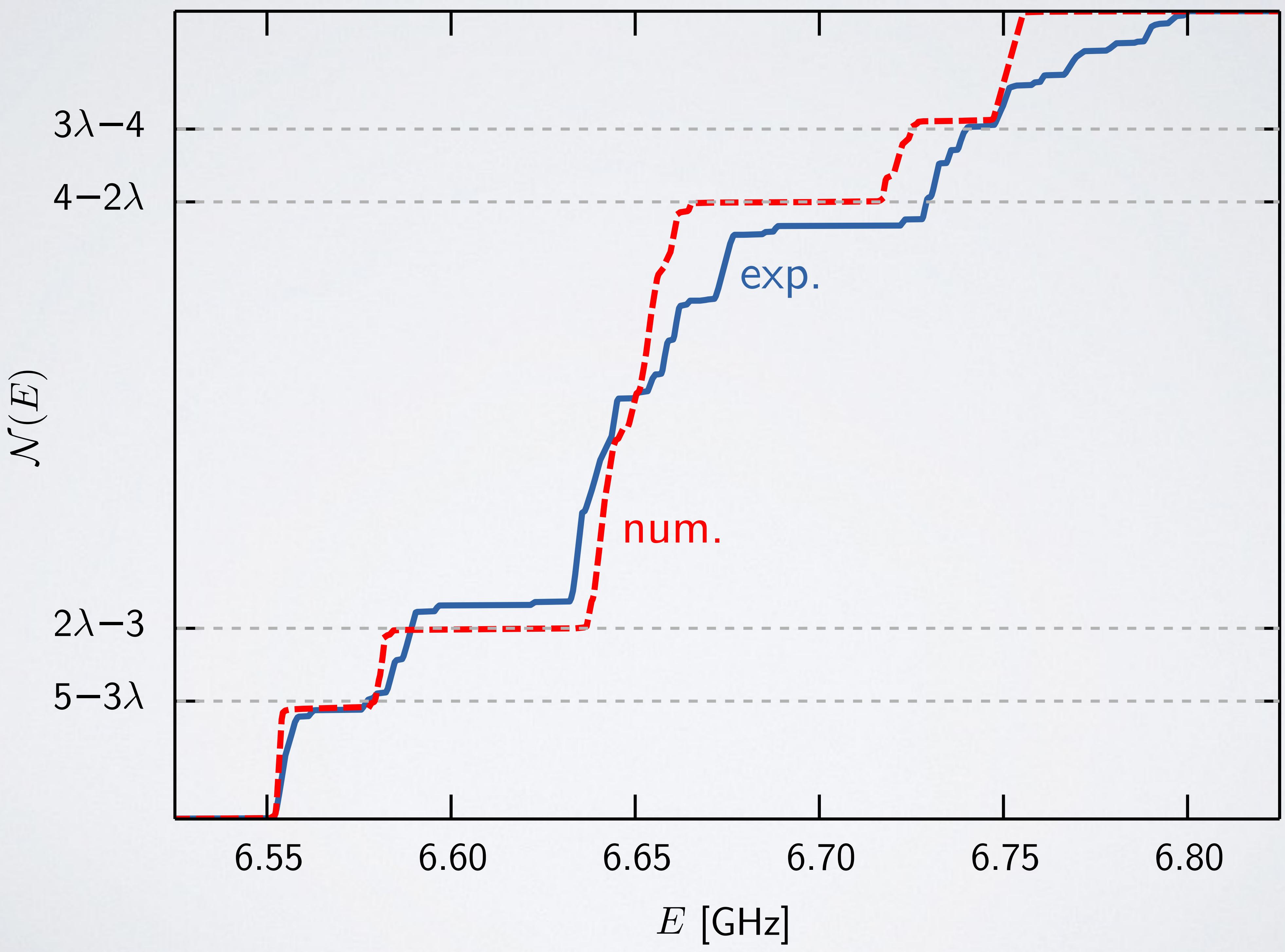
EIGENSTATES



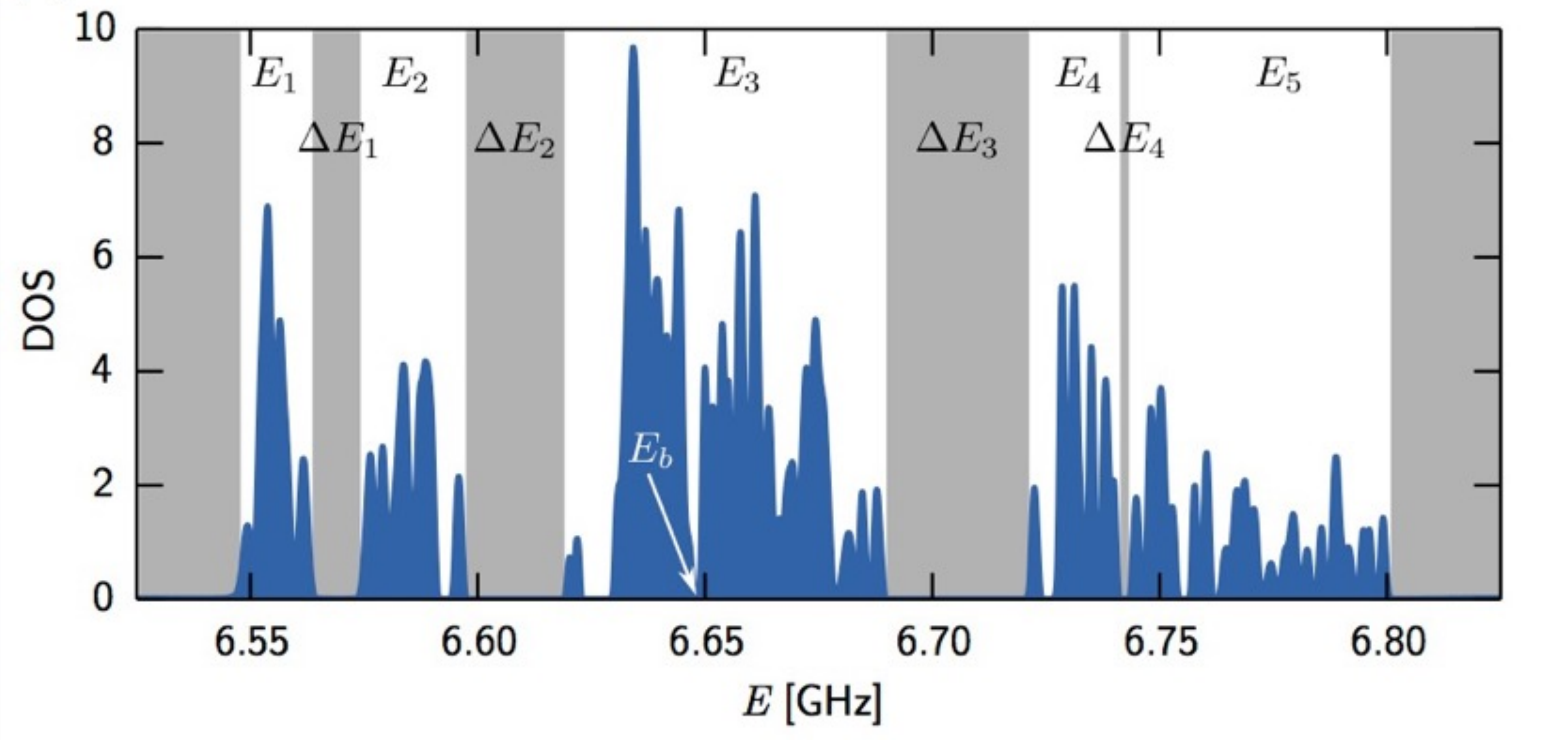
(INTEGRATED) DOS



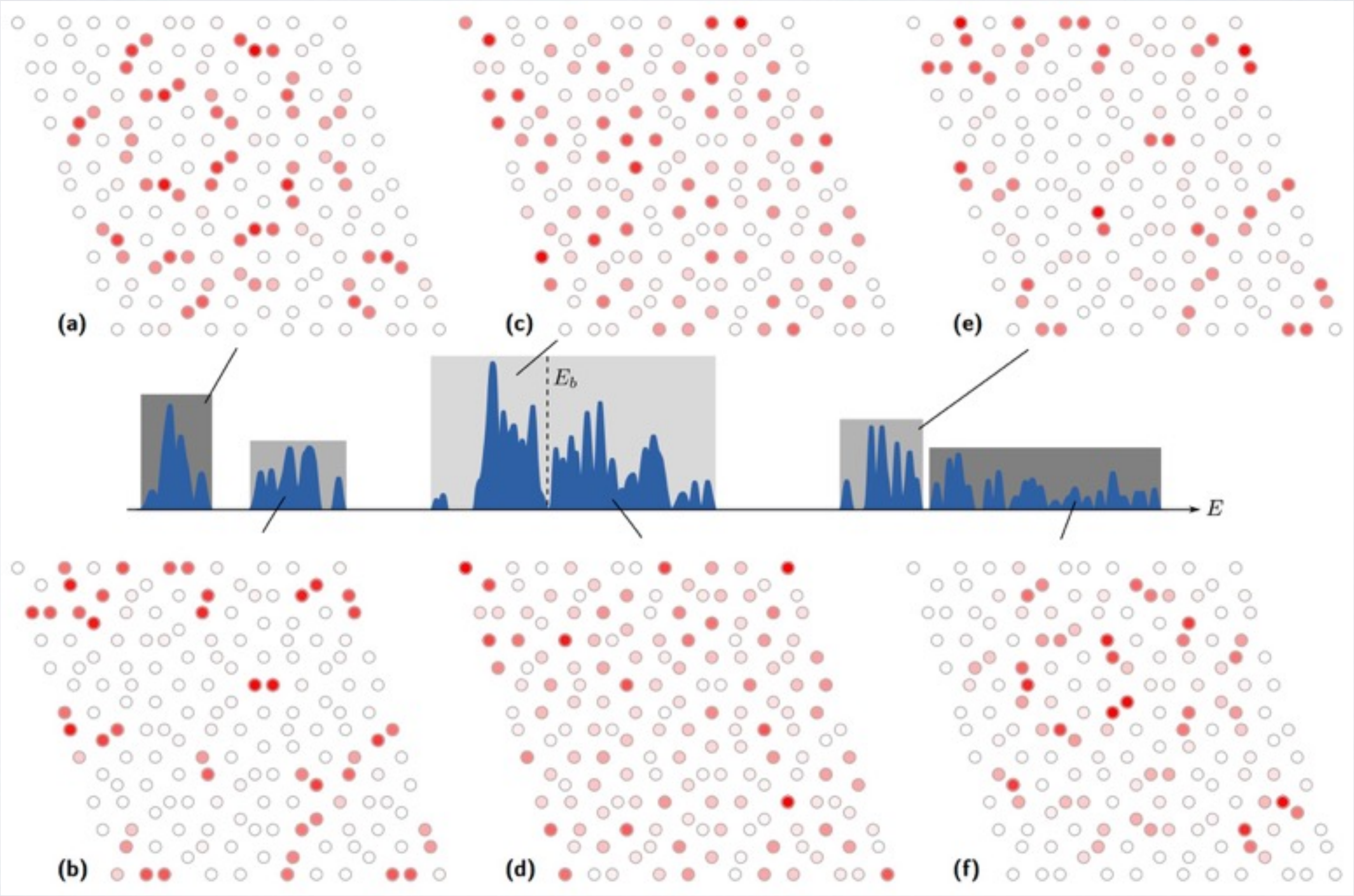
(INTEGRATED) DOS



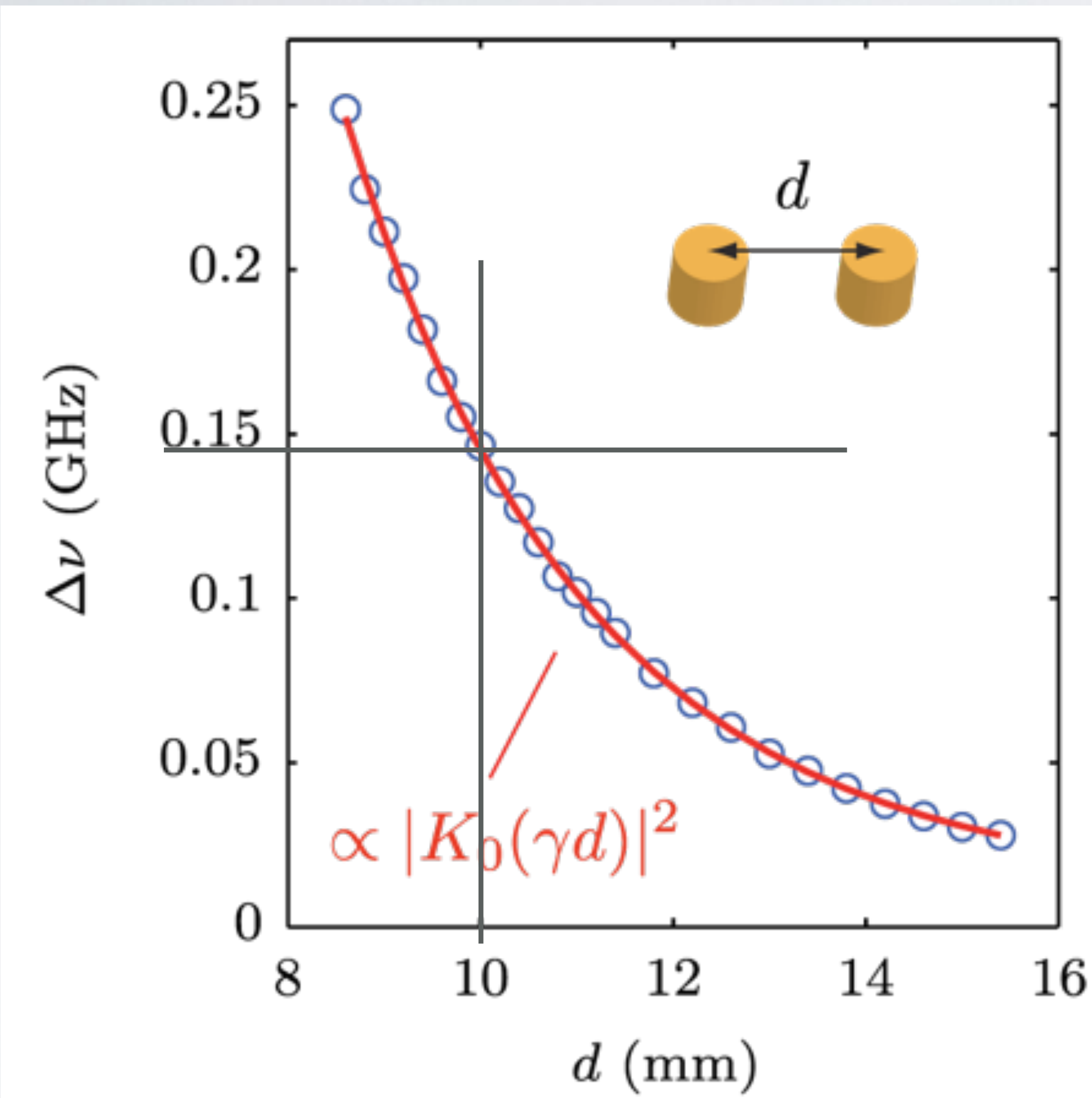
(INTEGRATED) DOS

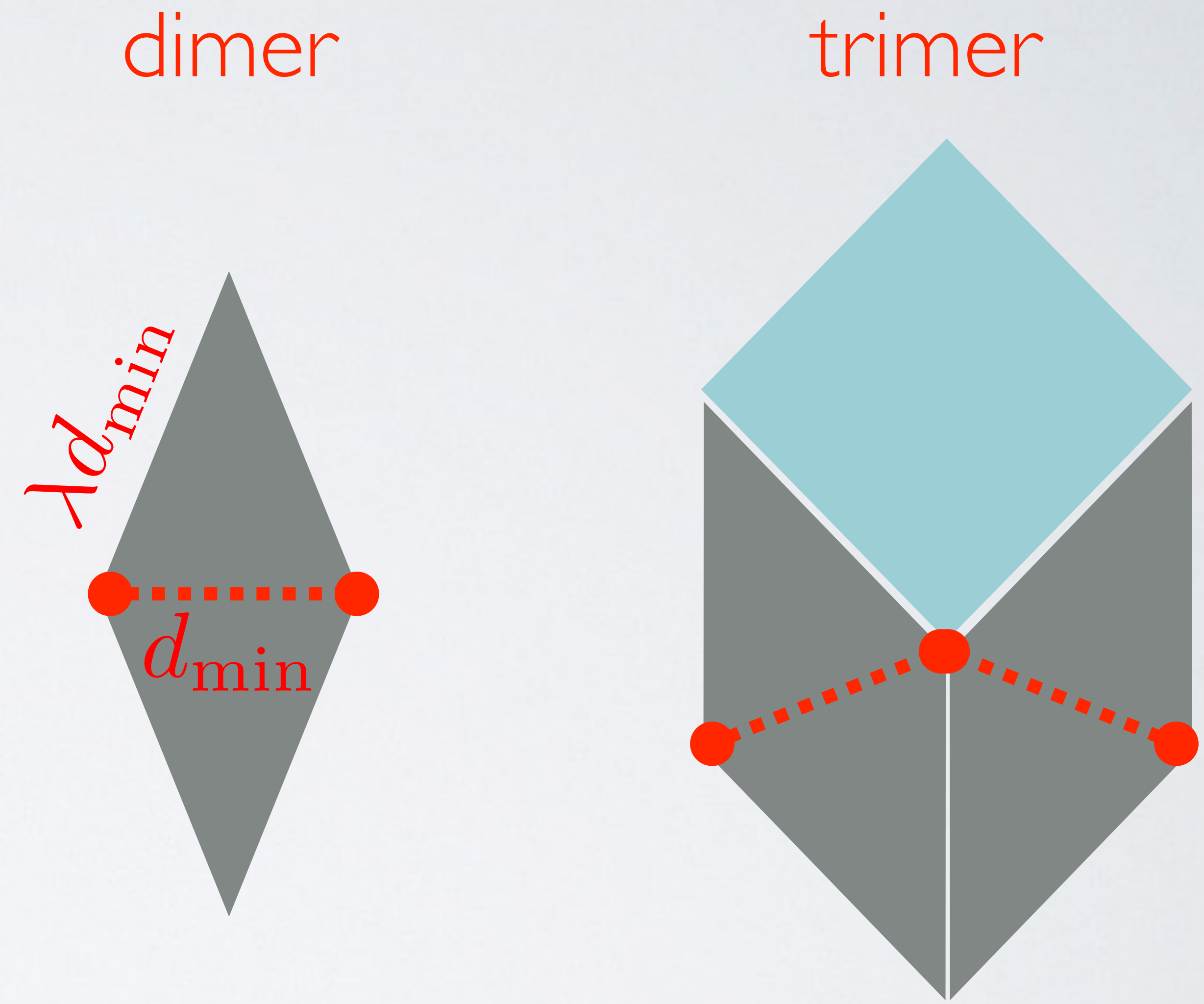


BAND WAVEFUNCTIONS



DOMINANT COUPLING

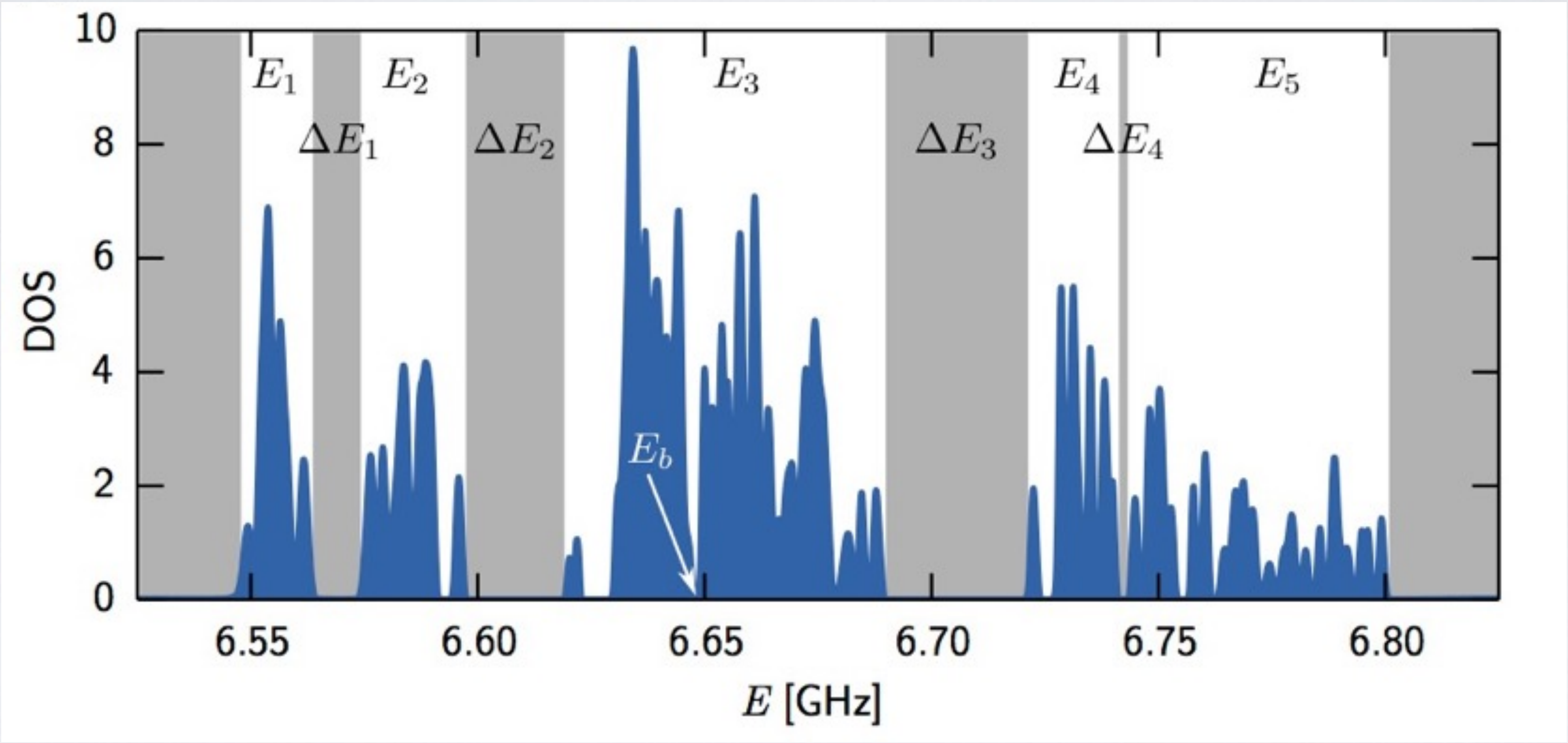




$$d_{\min} = 10\,\mathrm{mm} \Rightarrow t_{\max} \simeq 73\,\mathrm{MHz}$$

dominant coupling along the diagonal of the thin rhombus

BAND STRUCTURE



$$E_1 = E_b - \sqrt{2} t_{\max} \simeq 6.55\,\mathrm{GHz}$$

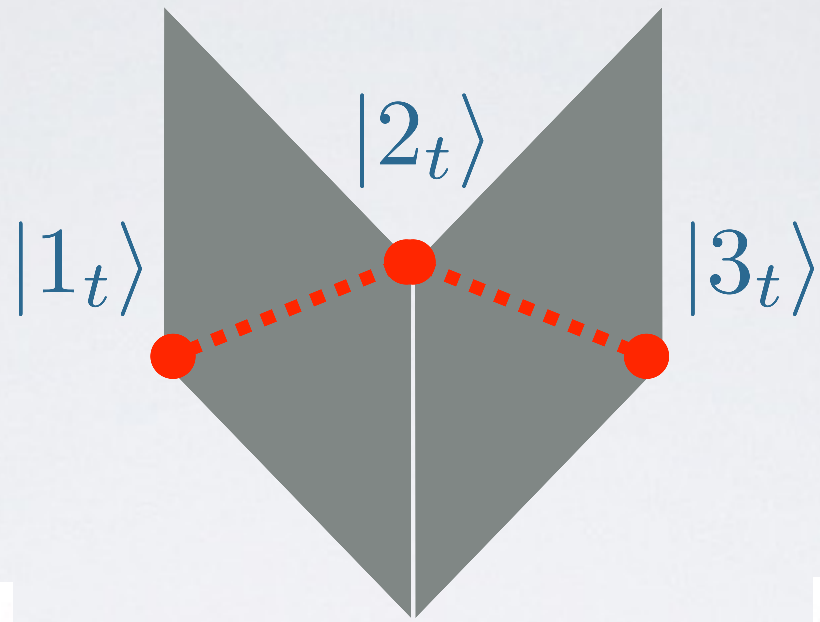
$$E_5 = E_b + \sqrt{2} t_{\max} \simeq 6.75\mathrm{GHz}$$

$$E_2 = E_b - t_{\max} \simeq 6.58\,\mathrm{GHz}$$

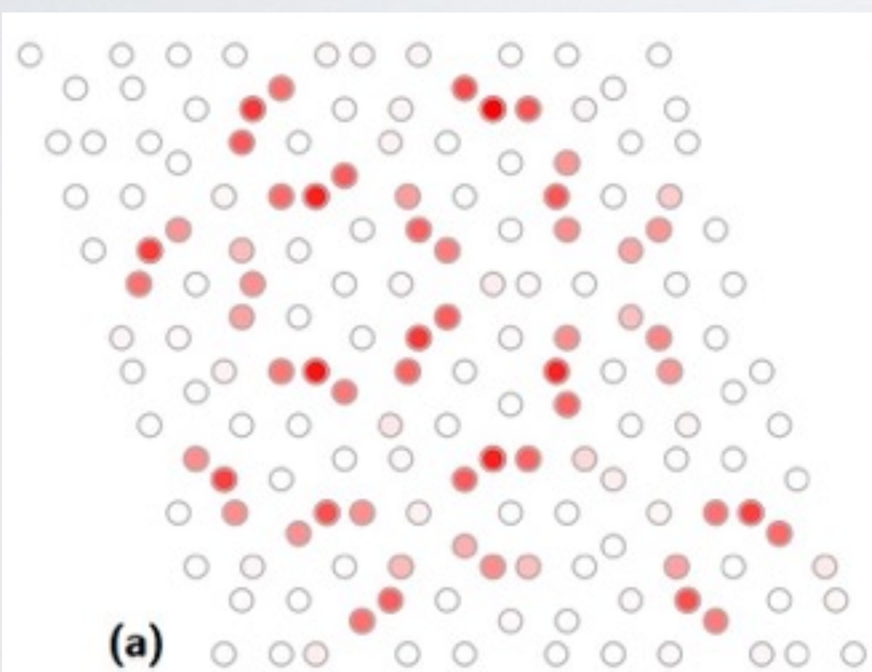
$$E_4 = E_b + t_{\max} \simeq 6.73\,\mathrm{GHz}$$

$$E_3 = E_b = 6.65\,\mathrm{GHz}$$

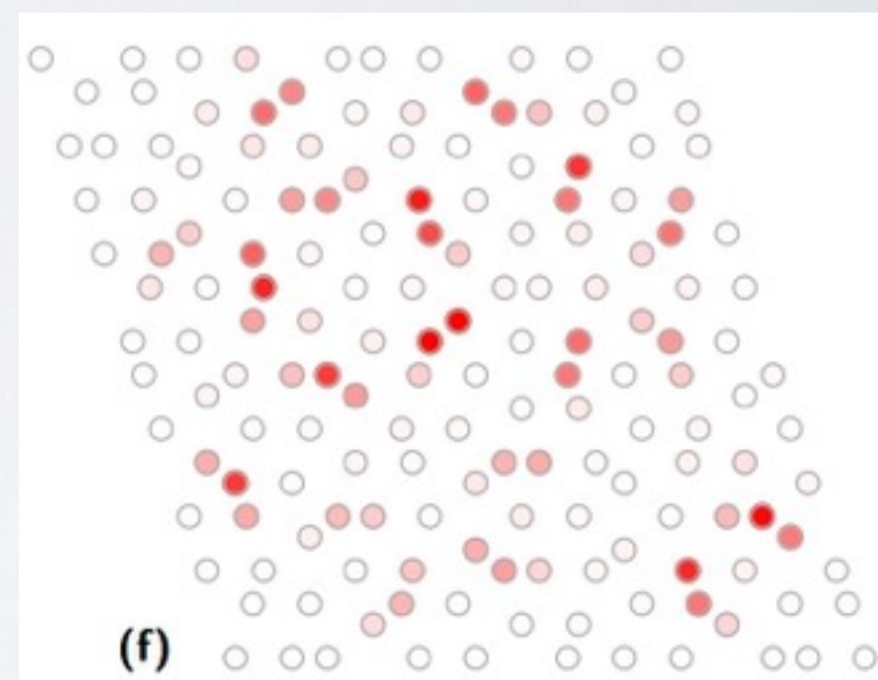
TRIMER MOTIF



$$E_1 = E_b - \sqrt{2}t_{\max}$$



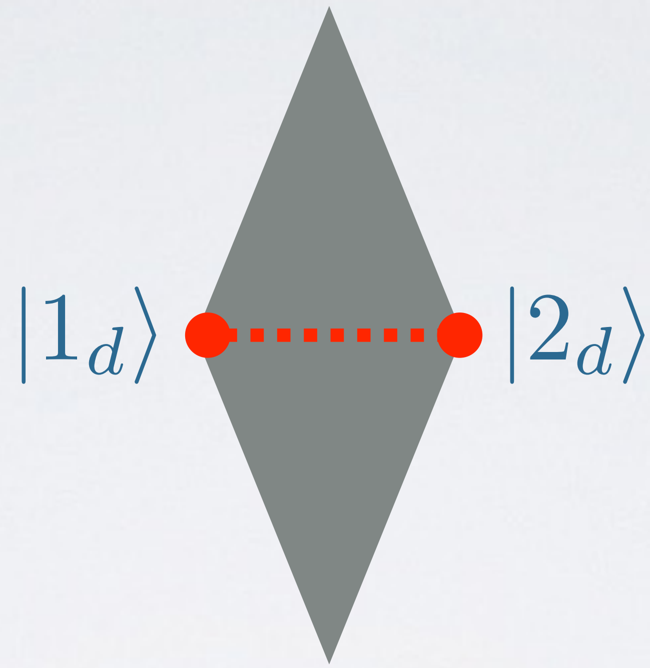
$$E_5 = E_b + \sqrt{2}t_{\max}$$



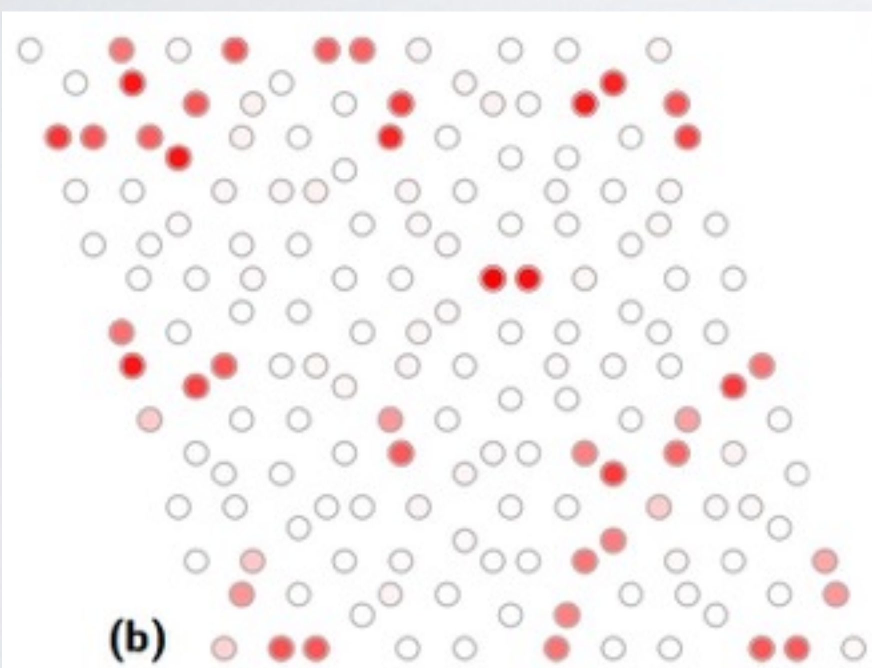
$$|\phi_1\rangle = |1_t\rangle - \sqrt{2}|2_t\rangle + |3_t\rangle$$

$$|\phi_5\rangle = |1_t\rangle + \sqrt{2}|2_t\rangle + |3_t\rangle$$

DIMER MOTIF

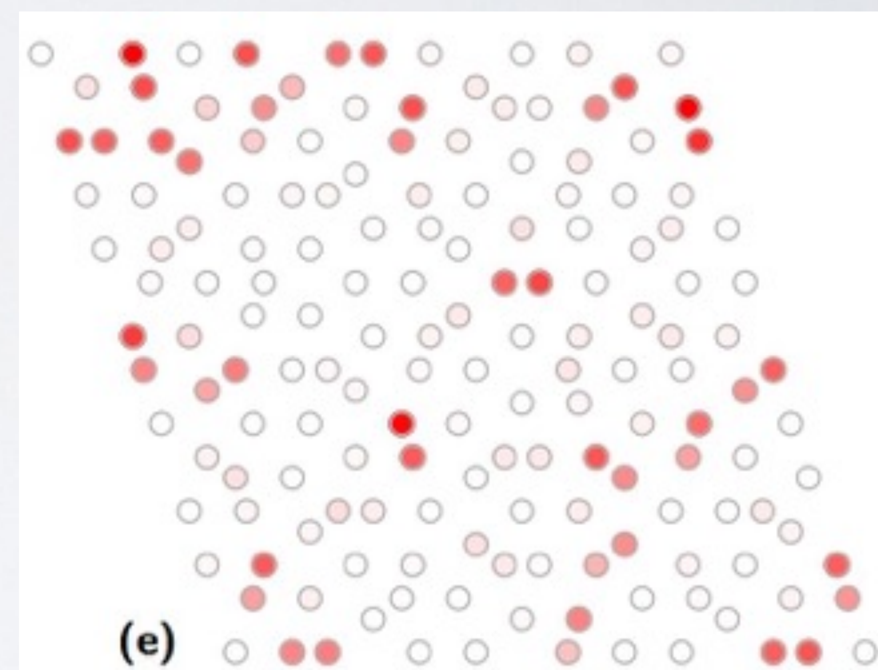


$$E_2 = E_b - t_{\max}$$



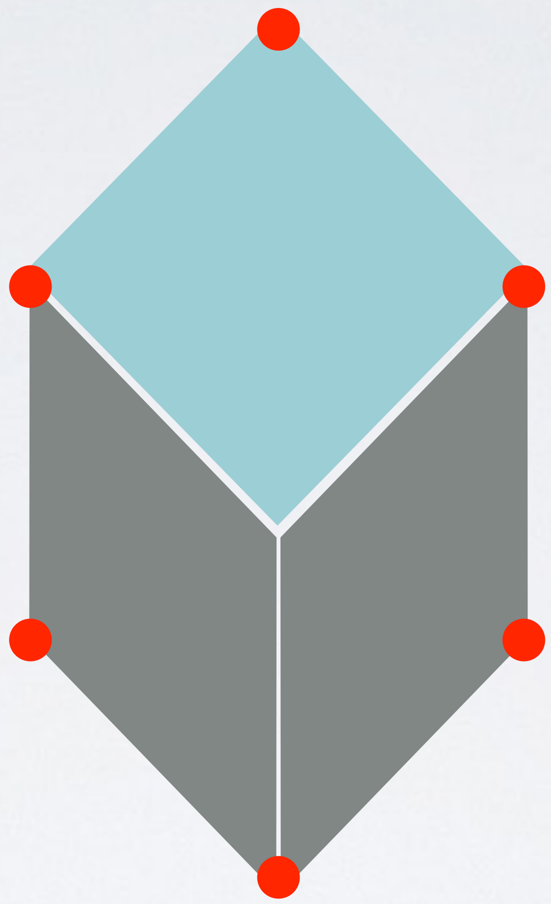
$$|\phi_2\rangle = |1_d\rangle - |2_d\rangle$$

$$E_4 = E_b + t_{\max}$$

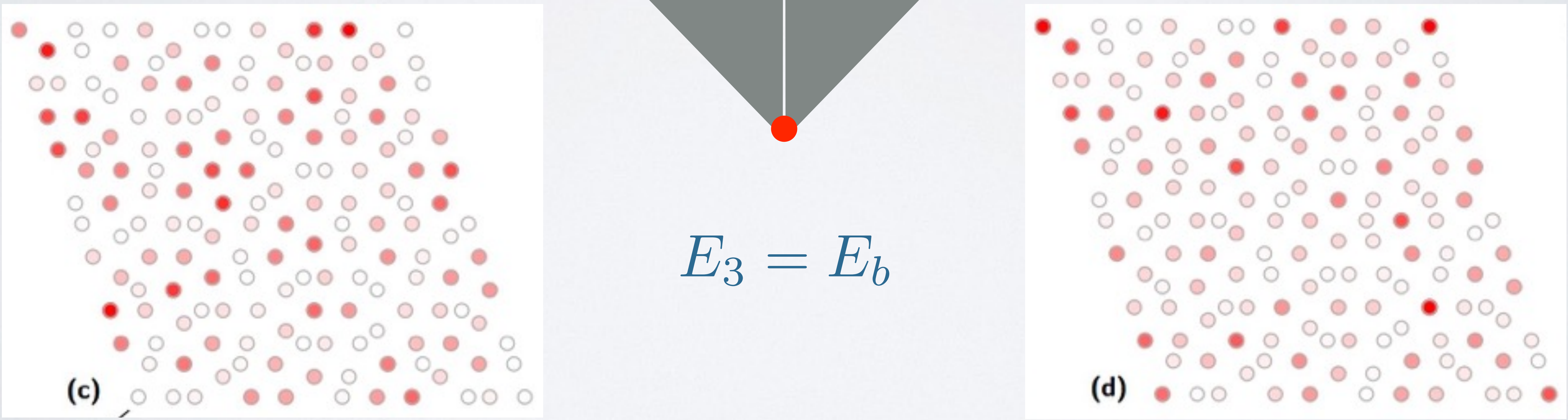


$$|\phi_4\rangle = |1_d\rangle + |2_d\rangle$$

'ISOLATED' SITES

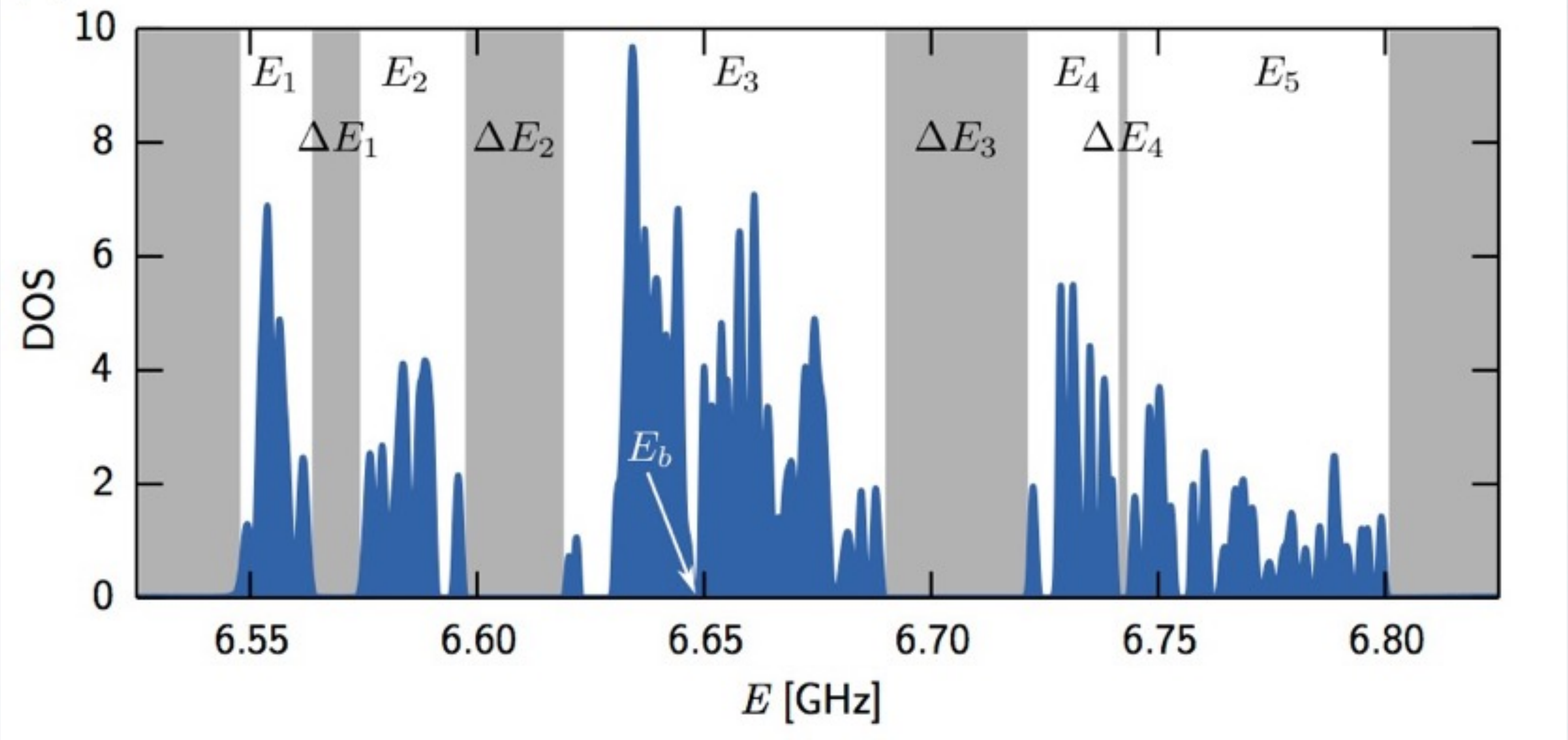


$$E_3 = E_b$$



$$|\phi_{3,a}\rangle = |1_t\rangle - |3_t\rangle \qquad |\phi_{3,b}\rangle = |1_s\rangle$$

BAND POPULATIONS

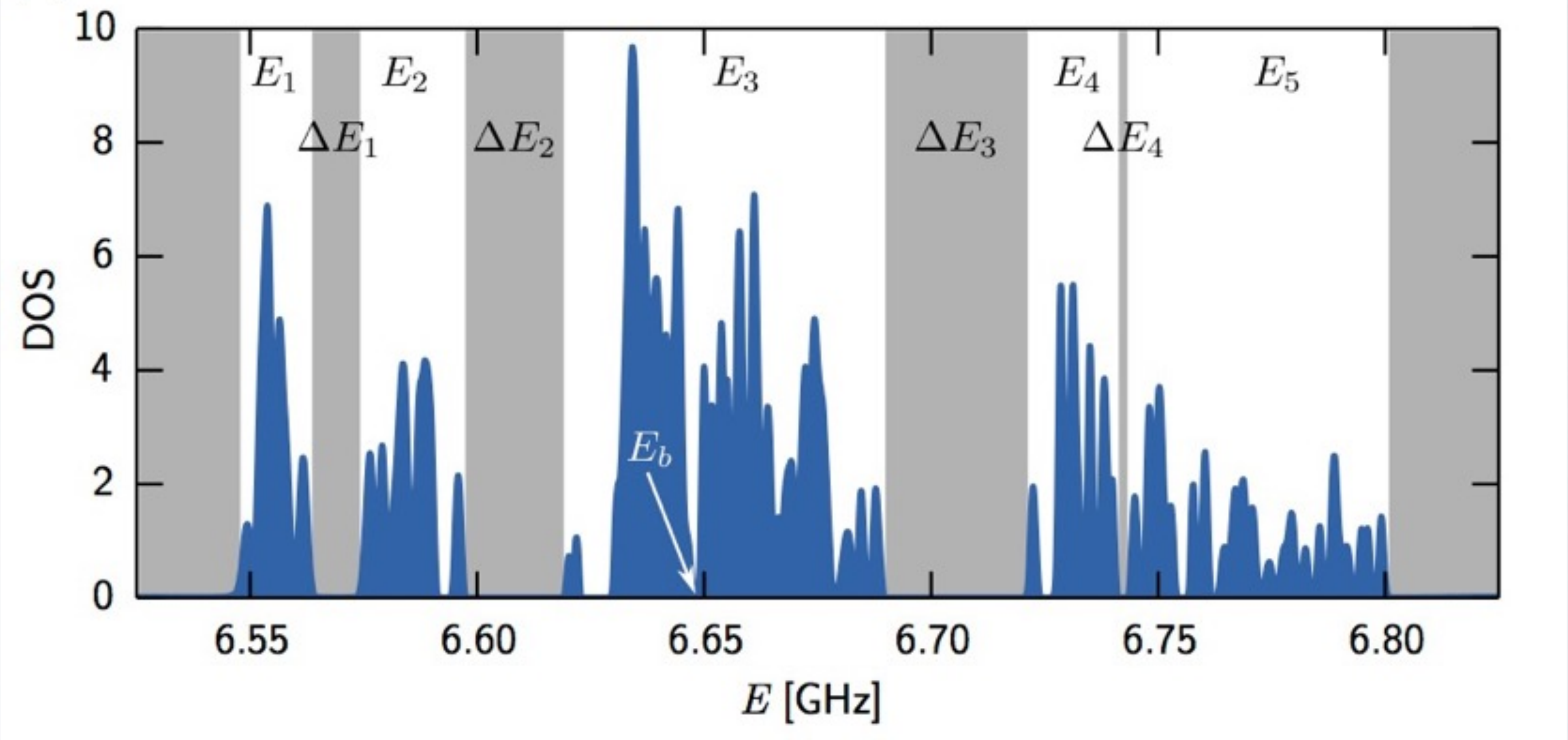


# of dimers: $\beta_1 = \beta_5 = 5 - 3\lambda$

# of trimers: $\beta_2 = \beta_4 = 5\lambda - 8$

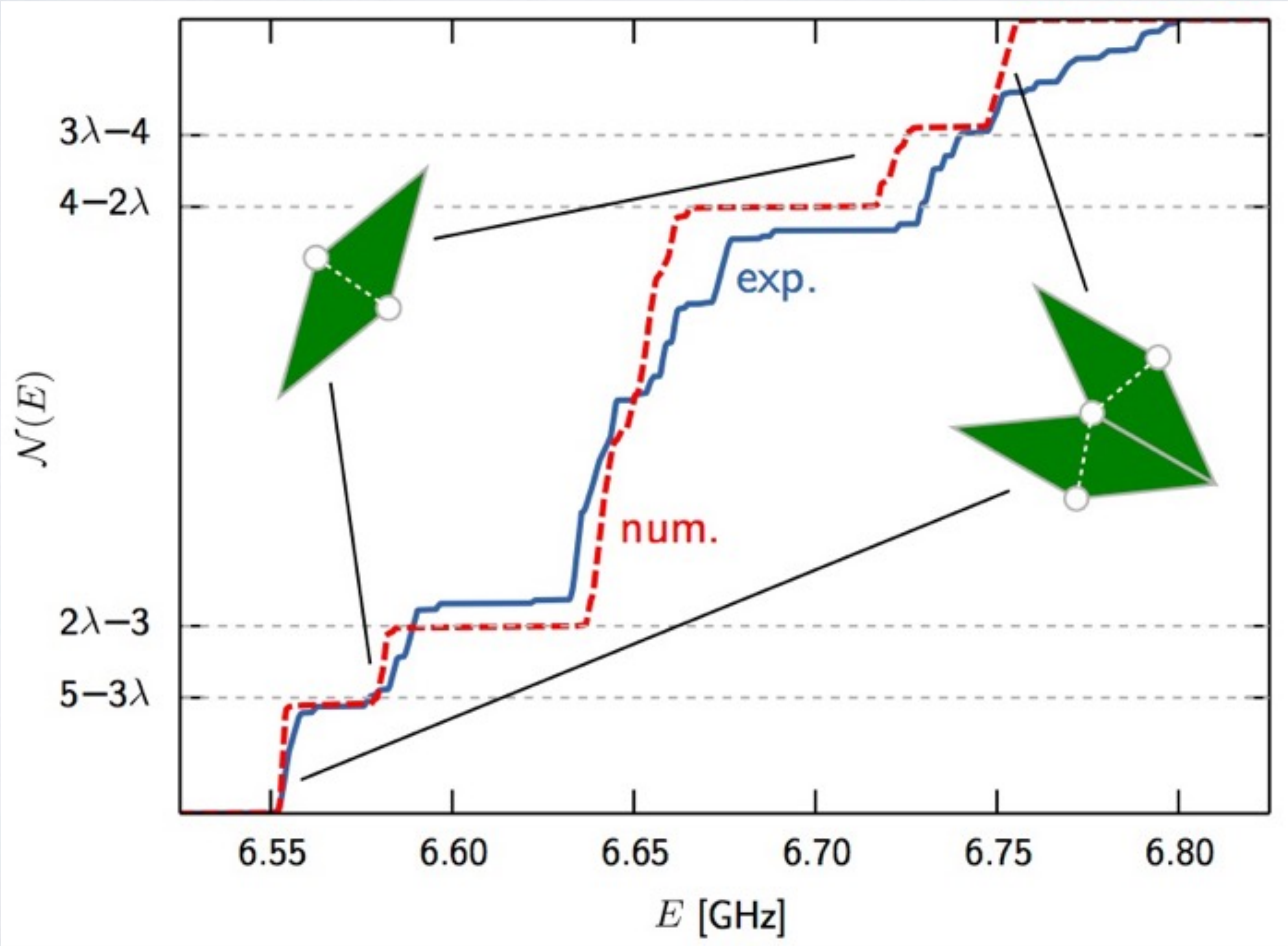
# of others: $\beta_3 = 1 - 2\beta_1 - 2\beta_2 = 7 - 4\lambda$

BAND POPULATIONS



$$\mathcal{N}(E) = \begin{cases} \beta_1 = 5 - 3\lambda, & E \in \Delta E_1 \\ \beta_1 + \beta_2 = 2\lambda - 3, & E \in \Delta E_2 \\ \beta_1 + \beta_2 + \beta_3 = 4 - 2\lambda, & E \in \Delta E_3 \\ \beta_1 + \beta_2 + \beta_3 + \beta_4 = 3\lambda - 4, & E \in \Delta E_4, \end{cases}$$

PHYSICAL PICTURE OF THE GAP LABELING



TAKE-HOME MESSAGE

- tight-binding model emulates with dielectric resonator lattices;
- flexible experimental platform giving access to DOS and eigenstates;
- experimental realization of a finite 2D quasicrystal with Penrose tiling;
- simple physical pictures of gap labeling and energy landscape.

FURTHER READINGS

- This talk: arXiv: 1411.1234.
- Artificial graphene: PRB **88**, 115437 (2013), PRL **110**, 033902 (2013).
- Disordered graphene & Boron Nitride: PRB **87**, 035101 (2013).
- Topological phase transition and edge states: Nature Com., **6**:6710 (2015), NJP **16**, 113023 (2014), PRL **110**, 033902 (2013).
- Dirac oscillator, Dirac gyroscope: PRL **111**, 170405 (2013), NJP **15**, 123014 (2013).
- Quantum search: PRL **114**, 110501 (2015).
- Topological state in 2D Lieb lattice: soon on arXiv.